Getting to the Core: Inflation Risks Within and Across Asset Classes*

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Abstract

Decomposing inflation into core and non-core components (e.g., energy) sheds new light on the nature of inflation risk and risk premia. While stocks have insignificant exposure to headline inflation in the U.S., their core inflation betas are negative and energy betas are positive. Conventional inflation hedges such as currencies and commodities only hedge against energy inflation risk but not the core. These hedging properties are reflected in the prices of inflation risks: only core inflation carries a negative risk premium and its magnitude is consistent both within and across asset classes, whereas the price of energy inflation risk is indistinguishable from zero. The relative contribution of core and energy inflation varies over time, which helps explain why the correlation between stock and bond returns appears to switch sign in the data. We develop a two-sector New Keynesian model to account for these facts.

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1 Introduction

Inflation is as a key macroeconomic factor and fundamental source of risk driving asset returns. Conventional wisdom holds that fixed income securities incur losses in the face of inflation, but stocks, foreign currencies, and commodities maintain their values. Stocks are claims on real physical assets. Foreign currencies hedge inflation according to the purchasing power parity. Commodity prices themselves are important components of total inflation. Because investors fear inflation, they may accept lower returns for inflation-hedging assets (Chen et al., 1986). Yet empirical evidence of such a risk premium has been elusive, as have the inflation-hedging properties of supposedly "real" assets, notably stocks (e.g., Bekaert and Wang, 2010, Katz et al., 2016).

We argue that decomposing inflation into core and non-core components (with a particular focus on energy) is important as it sheds new light on the nature of inflation risks. First, core and energy inflation have sharply different statistical and economic properties. Second, inflation-hedging properties of conventional "real assets," such as stocks, currencies, and commodity futures, are largely confined to energy inflation, while they provide almost no protection against core inflation risk. Third, core inflation carries a significantly negative price of risk, while the risk price associated with energy inflation is in most cases indistinguishable from zero.

In the data, core inflation is much more stable and persistent than energy inflation. Core and energy inflation series have a very low correlation, despite both being highly correlated with headline inflation. Economically, core goods' prices have a substantially higher degree of rigidity than energy prices, and they are potentially driven by different supply and demand shocks. These distinctions, largely glossed over in the literature, can potentially lead to very different inflation risks manifested in asset prices.

Armed with this decomposition we revisit inflation-hedging properties of different assets. We examine 7 major asset classes: U.S. stocks, Treasury notes/bonds, agency bonds, corporate bonds, currencies, commodity futures, and real estate investment trusts (REITs). The broad coverage of assets is informative since investors often consider multi-asset-class allocations when it comes to managing inflation risks. Our estimates of headline inflation betas confirm, to some extent, the conventional view on inflation hedging. Fixed income securities have negative betas, while currencies, commodities and REITs have positive betas. Stocks' headline betas are mostly negative but often statistically insignificant. After decomposing headline inflation into core and energy, we find that assets' exposures to the two components are sharply different. The core betas are consistently significantly negative for all stock and REITs portfolios while the energy betas are positive. Treasuries and agency bonds have negative exposures to both core and energy inflation shocks. Corporate bonds have negative core betas and insignificant energy betas. Furthermore, the exposures of currencies and commodity futures to energy are positive but they are insignificant for core inflation. Therefore, the conventional view mixes the two distinct components of inflation, core and energy, in a way that potentially obscures their effects on asset prices. Stocks have unambiguously negative core betas, while the insignificant headline betas are due to the presence of energy inflation. Currencies, commodities, and REITs, often considered as inflation-hedging assets, only hedge against the energy inflation but not the core.

Given these risk exposures, we ask whether hedging against core and energy inflation is costly. To answer this question, we conduct cross-sectional asset pricing tests using both the 7 average portfolios in each asset class and a larger cross section of 35 test portfolios. The price of headline inflation risk is around zero and insignificant, which seems to indicate that hedging against inflation is free. However, differentiating core from energy inflation, we find that the core inflation risk has a significant negative price of risk and the price of energy inflation risk is positive but indistinguishable from zero. In other words, hedging against core inflation is costly, since assets negatively correlated with core inflation shocks earn a risk premium, while hedging against energy inflation is essentially "free."

In addition to the unconditional inflation risk exposures of these test portfolios, we examine how these exposures - and their prices - vary over time. We show that term spread is a useful conditioning variable for capturing time-variation in the price of core inflation risk. Splitting the sample allows us to further investigate the time-varying inflation risk exposures. Recent studies highlight the fact that stock-bond return correlation changed its sign at the turn of the century (Song, 2016; Campbell et al., 2019). Stocks, currencies, commodity futures, and REITs have energy betas that are significantly larger in the post-1999 subsample than before, while the changes in core betas are insignificant. Moreover, core inflation becomes less volatile after the 1980s while energy inflation fluctuates widely in the 2000s. The growing importange of energy in total inflation (relative to core) and its positive correlation with stock returns helps provides a potentially new explanation for why the correlation between bond and stock returns switches from positive to negative in the recent subsample.

We further take advantage of our multi-asset-class setting to estimate the price of inflation risks within each asset class. Strikingly, the magnitude of core inflation risk price is rather consistent across asset classes. Average returns of assets line up well with core inflation betas both within and across asset classes, but are essentially unrelated to betas with energy or headline inflation. Therefore, different asset classes imply a largely consistent cost of hedging against core inflation. We construct the factor mimicking portfolios for headline, core, and energy inflation using portfolios from each asset class. Only the average returns of core inflation mimicking portfolios behave similarly, while the average returns of headline and energy inflation mimicking portfolios are unrobust and switch signs for different asset classes.

The empirical results are robust to controlling for various consumption and other macroeconomic factors.[1] In a seminal paper, Fama (1981) proposed that low inflation is a proxy for high growth of real activity, leading to the negative inflation betas of stocks. Our results show that the core inflation is an important risk by itself, and it is not driven out by standard measures of real activity. This echoes the findings in the New Keynesian DSGE literature that argues inflation "dances to its own tune " (Smets and Wouters, 2007).

Why is core inflation risk different from energy? We explore two potential mechanisms empirically by looking at different categories of inflation. First, we use inflation measures constructed using goods and services whose prices are either "sticky" or "flexible" and find that they resemble core inflation and energy inflation, respectively, in both their risk exposures and risk premia. This fact is consistent with the idea that energy prices are fundamentally different from core because they are easily adjusted, while the latter exhibit considerable stickiness. Second, we look at the cyclical and acyclical parts of core inflation and find that only the cyclical part inherits the properties of core inflation. These results suggest that price stickiness and cyclicality of inflation are both important for understanding inflation risk.

In order to further interpret our empirical findings, we develop a two-sector New Keynesian model that rationalizes the stylized facts listed above: (i) stocks are negatively exposed to core inflation; (ii) Treasuries (and nominal bonds more generally) are negatively exposed to both core and energy inflation; (iii) currencies and commodity futures are positively exposed to energy inflation; (iv) core inflation carries a negative price of risk while the price of energy inflation risk is positive but potentially difficult to distinguish from zero due to the countervailing effects of energy demand and supply shocks.

Our model includes the minimum set of ingredients necessary to qualitatively account for our empirical findings. In a small-scale New Keynesian economy with an energy sector, households consume both core and energy goods. There is a continuum of varieties of core goods, and each variety is produced by a monopolistic firm that chooses to set the nominal price of the good. Firms face price stickiness, i.e., only a fraction of firms can adjust their prices freely. The desired markup fluctuates exogenously and is the main driver of core

[1] We consider consumption growth rate, durable consumption growth rate (Yogo, 2006), industrial production growth rate (Chen et al., 1986), payroll growth rate, unemployment growth rate, and the long run and short-run consumption growth news constructed by Hansen et al. (2008), unfiltered consumption growth (Kroencke, 2017), and capital share growth (Lettau et al., 2019).

inflation. This is a modeling device to capture the variation of inflation that is independent of other real macroeconomic and policy shocks. Energy goods are subject to energy supply and demand shocks and, importantly, face no price rigidity.

When a markup shock increases the cost of production, core inflation rises, core output drops, and thus the marginal utility of consumption increases. Therefore, core inflation carries a negative price of risk. Stocks, claims to the core output, are negatively exposed to core inflation. Naturally, Treasuries have negative betas with core inflation. Furthermore, energy supply and demand shocks have opposite effects on energy inflation, but both shocks are expansionary to core production when core and energy goods are complementary. Therefore, an increase of energy consumption implies a higher consumption in core good. Since a high energy price combines the effect of negative supply and positive demand, their effects on core output, core inflation, and thus the stochastic discount factor are opposite in their signs. As a result, energy inflation carries a price of risk indistinguishable from zero. When the demand is the major driver of energy inflation, the model implies positive energy betas for stocks and currencies and negative betas for Treasuries.

Related Literature A large body of literature studies the inflation-hedging properties of financial assets (Fama and Schwert, 1977; Bekaert and Wang, 2010). Stocks have negative correlation with unexpected inflation, thus being poor inflation hedges. Fama (1981) argues that the negative correlation is caused by real activity that is correlated positively with stock returns and negatively with inflation. Katz, Lustig, and Nielsen (2017) propose that stock investors respond slowly to inflation. Nominal bond returns negatively covary with inflation. Commodity futures returns are positively correlated with inflation (Gorton and Rouwenhorst, 2006).

The cost of inflation hedging, a.k.a. the inflation risk premium, has been of great interest. In the stock market, Chen, Roll, and Ross (1986) document a marginally negative price of inflation risk. Boons et al. (2019) show that inflation risk is priced in the inflation-beta-sorted stock portfolios but its sign changes from negative to positive after 2000. Inflation risk is a common feature in term structure models that distinguish between real and nominal bonds (Ang, Bekaert, and Wei, 2008). While most of this work has focused on the headline inflation, one important exception is Ajello, Benzoni, and Chyruk (2019) who incorporate both core and "crust" inflation components into an affine term structure model. In currencies, Hollifield and Yaron (2003) found little evidence of an inflation risk premium. In contrast to much of this work, we utilize a large cross section of multi asset classes including currency portfolios (Lustig et al., 2011, 2014; Menkhoff et al., 2017; Verdelhan, 2018) and commodity futures portfolios (Bakshi, Gao, and Rossi, 2019) and find largely consistent magnitudes of inflation risk premia across them.

Among theoretical studies of inflation risks and asset prices, several equilibrium models with an endowment economy can quantitatively match inflation, term structure and stock returns (Buraschi and Jiltsov, 2005; Wachter, 2006; Bansal and Shaliastovich, 2012). In Bansal and Shaliastovich (2012), the inflation premium is due to the negative effect of inflation on future long-run real growth. Eraker et al. (2016) further find the inflation-growth effect is more pronounced in the durable goods sector, and durable stocks are more exposed to inflation risks. Song (2016) estimates a regime-switching model and variations in the cyclical properties of inflation and its premium. To explain the new facts, we propose a New Keynesian model with production and an energy sector and study it analytically. Our model highlights the role of markup shock as the major source of priced inflation risk. This is consistent with Smets and Wouters (2007) that emphasize that inflation is mostly driven by markup shocks in a quantitative NK-DSGE model. Kung (2015) builds a New Keynesian model with an endogenous interaction between inflation and real growth in order to account for several stock and bond price puzzles. Campbell, Pflueger, and Viceira (2019) estimate a New Keynesian model and show that time-varying stock-bond correlation is driven by monetary policy regimes. Weber (2015) shows that firms facing stronger price rigidity earn a premium. Bodenstein et al. (2008) use a model similar to ours to study optimal monetary policy.

This paper also contributes to studying the interaction between commodities and other asset prices (Ready, Roussanov, and Ward, 2017a,b; Ready, 2017, 2018). Commodities are not only an asset class but also a source of macroeconomic risk in their own right. We explicitly model and analyze the role energy commodities in driving inflation risk. There is a ongoing debate in the literature regarding the relative importance of different types of shocks (e.g. supply vs. demand) in driving the prices of key energy commodities such as crude oil (Kilian, 2009; Baumeister and Hamilton, 2018). We contribute to this literature by bringing in asset prices, which move differently in response to shocks to energy demand and supply, as an additional source of identifying variation.

2 Empirical Analysis

2.1 Data and Descriptive Statistics

2.1.1 Inflation

We use the consumer price index (CPI) and its components from the U.S. Bureau of Labor Statistics as inflation measures. The CPI can be decomposed into three components: core

(CPI less food and energy), food, and energy. The expenditure categories in core include shelter, household furnishings and operations, apparel, transportation, medical care, recreation, education and communication, alcoholic beverages, and others goods and services (tobacco, personal care, etc). The sample is at the quarterly frequency from 1963Q2 to 2019Q4.

The Panel A of Table 1 reports the summary statistics of inflation. The headline inflation (CPI) and its three components have similar average of about 4 percent per annum over the sample. Core, food, and energy differ greatly in their volatility and persistence. Core inflation has a low volatility and a high persistence, with a standard deviation of 2.66 percent per annum and an autocorrelation of 0.79. In contrast, energy inflation is much more volatile with a standard deviation of 19.52 percent per anuum and exhibits little persistence. The food inflation stands between core and energy inflation in both volatility and persistence. The large difference in their persistence can be attributed to the different degree of price rigidity in those goods. Core goods and services, such as apparel, shelter, medical care, feature stronger price rigidity, while energy prices are quite flexible.

Panel B of Table 1 reports the relative weights of the three components in the headline inflation. These weights are obtained by regressing the headline inflation onto the three components. Core inflation accounts for 71 percent of headline inflation, food accounts for 20 percent, and energy accounts for the least, only 9 percent of headline inflation. Although core inflation accounts for the largest portion, energy inflation is much more volatile and substantially drives headline inflation as well.

In Panel C of Table 1, we examine the correlation structure of headline inflation and the three components. All three components are fairly correlated with headline inflation (core 0.80, food 0.60 and energy 0.69). However, the correlations across the three components are much lower. Energy inflation is correlated with neither core (0.20) nor food inflation (0.17), while food and core inflation has moderate correlation of 0.44.

To summarize, the three components of headline inflation have distinct volatility and persistence, and they are not correlated with each other, especially the core and energy components. Because energy inflation exhibits a stark contrast with the core inflation, we focus on the core and the energy part of the noncore inflation, and leave out the food part for parsimony.

2.1.2 Asset Returns

We use test portfolios from a wide and standard asset classes: stocks, Treasuries, agency bonds, corporate bonds, currencies, commodity futures, and REITs. We first consider an average portfolio in each asset class. An average portfolio for stock, agency bond, commodity future, and REITs is constructed using the respective market index. The average Treasury and corporate bond portfolio returns are the average of the cross-sectional portfolios below. The average currency portfolio is the equal-weighted average of the six interest rate sorted carry portfolios.

We examine a wider cross section by including a set of portfolios in each asset class. These assets include 5 industry stock portfolios: consumer, manufacturing, high tech, health, and others, 7 fixed-term Treasury portfolios, 4 maturity sorted agency bond portfolios, 4 maturity sorted corporate bond portfolios, 6 interest rate sorted currency carry portfolios Lustig et al. (2011) and the dollar carry portfolio (Lustig et al., 2014), 5 commodity future portfolios of major categories (livestock, industrial metal, precious metal, energy, and agriculture), and 3 REITs portfolios (equity, mortgage, and hybrid). These data are obtained from different sources: stock returns are from Ken French's website; Treasury returns are obtained from CRSP; agency bond returns are calculated based on ICE BofA agency index; corporate bond are from Barclays; currency data are downloaded from Datastream; commodity returns are constructed from the GSCI index; and REITs returns are obtained from CRSP Ziman REITs indexes. Data for different asset classes have different starting dates. The longest data go back to 1963 for stocks and Treasuries. Corporate bond data start from 1973, REITs data start from 1980, and currency data start from 1983. Commodity future returns start from different dates: 1970 for livestock and agriculture, 1973 for precious metal, 1977 for industrial metal, and 1983 for energy.

The summary statistics of the average portfolios in each asset class and the cross section of test portfolios are shown in the first two columns of Table 2 and 3. Notably, assets in different asset classes are highly dispersed in average excess returns. For example, the 5 stock portfolios have an average excess return of around 6-9 percent, and the 4 corporate bond portfolio excess returns are 2-4 percent on average. Treasury excess returns are smaller from 1 to 3 percent. Currency excess returns are dispersed, from -1.81 percent for the lowest interest rate portfolio to 5.56 percent for the highest interest rate portfolio, and the dollar carry portfolio has an average return of 5.34 percent. Commodity futures' excess returns are dispersed as well, from about zero for agriculture and above 7 percent for energy. The equity and hybrid REITs have excess returns even higher than stocks and the mortgage REIT's average excess return is about 5 percent. Stocks, commodity futures and REITs returns are the most volatile, while Treasury returns are the least volatile.

2.1.3 Inflation Shocks

To study the inflation risk, we extract the unexpected component in headline, core, food, and energy inflation from the following VAR system.

$$Y_t = c + AY_{t-1} + \varepsilon_t, \tag{1}$$

where Y_t includes the vector of headline, core, food, and energy inflation, and the risk-free rate, price-dividend ratio of the aggregate stock market portfolio, and the output gap. The first four elements of ε_t are extracted as the innovations to the four inflation variables in the vector of Y_t. Figure 1 plots the time-series of innovations to the four inflation variables.

The headline inflation shock combines the variation of the three elements. The large spikes in headline inflation shocks are generally driven either by energy or food inflation. For the episodes of 1970s and 1980s, the core inflation is quite volatile. Before mid 1980s, core inflation tracks the headline inflation closely. After mid 1980s, core inflation is much less volatile than the headline. Food inflation also becomes less volatile after the mid 1980s, but overall it is still more volatile than core inflation. Energy inflation is a magnitude more volatile than other inflations, especially after the late 1990s.

2.2 Inflation Hedging: Core and Energy

Our baseline regression is specified as follows:

$$r^e_{i,t} = \alpha_i + \beta^i_\pi \varepsilon_{\pi,t} + u_{i,t}, \tag{2}$$

where $r^e_{i,t}$ is the realized nominal return of asset i in excess of the nominal risk-free rate. β^i_π represents how much asset i's excess return changes with the shocks to inflation and its components. The shocks are extracted from the VAR of equation (1).

The risk-free rate incorporates changes in inflation expectation, but the realized inflation surprise is not included in the pre-determined risk-free rate. Therefore, a perfect inflation hedging asset should one-to-one move with the inflation surprise, i.e., $\beta^i_\pi = 1$. If β^i_π is significantly positive but less than 1, the asset is an imperfect inflation hedge.

2.2.1 The Average Portfolios

We start with the 7 average portfolios on the left hand side and the headline inflation shock on the right hand side. Panel A of Table 2 displays the results. The loadings of stocks on the headline inflation are negative but insignificant. Treasuries, agency bonds, and corporate bonds all have significantly negative headline inflation betas. Currencies and commodity futures hedge against headline inflation. The coefficient for the currency portfolio is close to 1, which suggests that the foreign currency is a perfect hedge. The commodity future return moves much more than the headline inflation with a coefficient of 8.59. REITs' headline beta is close to 0 and statistically insignificant. The results in Panel A are consistent with the conventional wisdom that currencies and commodity futures are inflation-hedging assets.

However, a different picture emerges when we examine core and energy inflation separately. In Panel B of Table 2, we report regression results with core and energy inflation shocks on the right hand side. Stocks load negatively on core and positively on energy inflation, both statistically significant. This result sheds new light on the ambiguous inflation-hedging property of stocks in the literature, as in Panel A of Table 2. The ambiguity in the sign of the stock's inflation betas is due to the mixture of core and energy inflation. The negative core beta and the positive energy beta add up to an insigificant loading on the headline inflation. Treasuries, agency bonds, and corporate bonds have negative betas with both core and energy inflation. The REITs' core and energy betas are similar with stocks. Currencies and commodity futures' hedging properties against headline inflation mainly come from the energy component, while their core betas are negative and insignificant.

The sharp contrast on inflation hedging properties between the two panels in Table 2 show the importance of decomposing the headline inflation into core and energy components. The average stock, currency, commodity future, and REITs have core and energy betas with opposite signs. The conventional wisdom that stocks, currencies and commodity futures are real assets is incomplete: they only hedge against energy inflation. A long position in none of these 7 asset classes can hedge against the core inflation.

2.2.2 The Full Cross Section

We next run regression (2) for the full set of 35 test portfolios and present the results in Table 3. The results are similar with Table 2. Within stocks, the five industry portfolios have heterogeneous exposures. They are negatively exposed to headline inflation except for manufacturing, but only two betas are significant. On the contrary, stocks' core betas are all unambiguously negative and statistically significant, while their energy betas are positive and mixed in statistical significance. All Treasury and agency bond portfolios have negative exposures to headline, core, and energy inflation. Corporate bonds load negatively on core inflation and their energy inflation betas are mixed in sign. One possible reason is that the default risk is eased when the energy price is high and economic conditions are good.

Carry trade portfolios mostly load negatively on the core inflation and the loadings decline with interest rate. High-interest-rate currencies load more negatively on core inflation and more positively on energy inflation. The dollar carry portfolio has a much more negative beta with respect to core inflation than the average currency portfolio. This observation implies that the curreny exposure to core inflation shocks depend on the level of interest rate. In contrast, betas with repsect to energy inflation do not exhibit strong conditional patterns. For commodity futures, energy naturally has a large exposure to energy inflation and so do other commodity future portfolios. But they do not hedge against the core inflation, with an exception of agriculture whose core beta is positive but statistically insignificant. The three REITs portfolios all have highly significantly negative core betas. The magnitude of these core betas are comparable to those of stocks. REITs are positively exposed to energy inflation, though only that of the equity REITs is statistically significant.

The findings confirm the conclusion we draw with the 7 average portfolios: Exposures to core and energy inflation are fundamentally distinct, especially for the conventional "real" assets.

2.3 Getting to the Core: The Inflation Risk Premium

In the previous section, we show that different asset classes have different exposures to core and energy inflation shocks. In this section, we further explore the cost of hedging against inflation, or the price of these inflation risks. We find that the price of risk of headline and energy inflation are indistinguishable from zero. However, getting to the core inflation, it carries a sizable negative price of risk.

2.3.1 Inflation Risks Across Asset Classes

Our analysis is based on a factor model of average returns,

$$E(r_{i,t}) = \beta_i' \lambda,$$

where λ is the vector of prices of risks. We run a Fama-MacBeth cross-sectional regression of average returns onto asset betas to estimate the price of risks and report the results in Table 4. The price of headline risk is statistically insignificant.With core and energy inflation as separate risk factors, core inflation carries a negative price of risk -1.07 that is significant at 99%, while the price of energy inflation risk is positive but insignificant. Assets with higher average returns tend to load more negatively on and get more hurt by core inflation. The cross-sectional fit is superb with an R^2 of 0.99. With 35 test portfolios, we utilize more variations in both average returns and asset betas and find a similar price of core inflation risk -1.06 and an even larger t-statistic. The two sets of test portfolios lead to similar estimates of the price of core inflation risk. In the appendix, we report robust results using the GMM method.

The price of risk estimates uncover the second source of difference between core and energy inflation, the cost of exposure. Investors require a compensation of 107 basis points of excess return per annum if an asset's increases one unit of exposure to core inflation in absolute value. Notably, nearly all assets in our universe have negative betas and carry core inflation premia. Compensation for energy inflation exposure is the opposite in sign and statistically insignificant. From a hedging perspective, hedging against core inflation is costly, while the cost of hedging against energy inflation is indistinguishable from zero.

To visualize the result, Figure 2 plots the average excess returns of the 7 average portfolios (the upper panel) and 35 portfolios (the lower panel) against their model predicted expected excess returns using headline inflation as a risk factor only. Both sets of portfolios have very poor model fit. Though the average excess returns for different asset classes vary substantially, the model predicted returns center around zero. This is reflected in the small price of risk estimates for both sets of portfolios.

In Figure 3, we plot the average excess returns against model but using both core and energy inflation as risk factors. The cross-sectional fit improves substantially. For 7 average portfolios, the average realized returns and model implied returns line up perfectly. Even with 35 portfolios, these returns line up nicely with an R^2 of about 0.8. The sharp contrast highlights the value our decomposition in understanding the average returns both within and across asset classes. Negative exposures to core and positive exposures to energy inflation are rewarded with additional returns. To see it more clearly, in Figure 4 and 5, we plot the cross-sectional relation between average excess returns and headline, core, and energy inflation betas for the 35 portfolios. Headline betas do not explain average return differences at all: stocks, bonds, and REITs have similar betas but their average returns differ. The core inflation betas line up well negatively with the average excess returns. The average stock and REITs portfolios have large, negative core betas and the highest returns. Treasuries, agency bonds, and corporate bonds have sizable negative betas and modest returns. Currencies and commodity futures have small exposures and their returns are relatively low. The pattern between average excess returns and energy inflation betas is quite noisy as well.

This figure shows the importance of using test portfolios from multiple asset classes. While core and energy betas within the same asset class differ, betas across asset classes are more dispersed and in line with their average excess returns. Dispersed betas improve the power of the statistical test.

2.3.2 Inflation Risks Within Each Asset Class

In the analysis above, we include 35 test portfolios from 7 asset classes to maintain a roughly equal number of test portfolios in each asset class so that none of the asset classes dominate in the price of risk estimates. The limitation of the choice is that the number of test portfolios are small when we examine inflation risks within each asset class. Therefore, we expand the test portfolios in each asset class. The expanded test portfolios include 35 stock portfolios, 19 Treasury portfolio, 6 agency bond portfolios, 8 corporate bond portfolios, 17 currency portfolios, 8 commodity future portfolios, and 11 REITs portfolios. The 35 stock portfolios include 17 industry portfolios and 18 double-sorted portfolios on size and book-to-market, investment, and profitability. The 19 Treasury portfolios include 7 fixed term portfolios and 12 maturity-sorted portfolios (<6M, 6-12M, 12-18M, 18-24M, 24-30M, 30-36M, 36-42M, 42-48M, 48-54M, 54-60M, 60-120M, >120M). The 6 agency bond portfolios are sorted on maturity: 1-3, 3-5, 5-7, 7-10, 10-15, and >15 years. The 8 corporate bond portfolios are double sorted on credit rating (Aaa-Aa and A-Bbb) and maturity (1-3, 3-5, 5-10, >15 years). The currency portfolios include the 7 portfolios used in the previous analysis plus 4 value-sorted portfolios (Asness et al., 2013; Menkhoff et al., 2017), and 6 dollar beta sorted portfolios (Verdelhan, 2018). For commodity futures, we additionally examine the three main components of the precious metal: gold, platinum, and silver. We consider 8 additional REITs portfolios: unclassified, diversified, health care, industrial/office, lodging/resorts, residential, retail, and self-storage.

With the expanded set of test portfolios, we examine the price of inflation risks in each asset class. [2]Table 5 reports the estimates based on test portfolios from each asset class, the 7 average portfolios, and the full cross section of 35 test portfolios. Strikingly, using test portfolios from different asset classes, we obtain a largely consistent estimate of the price of core inflation risk around -1. In Figure 3, we do see that assets in different asset classes have a largely similar slope between average excess returns and core inflation betas. Therefore, the core inflation risk is priced consistently both within and across asset classes.

2.3.3 Inflation Factor Mimicking Portfolios

Both core and energy inflation are macroeconomic factors that are not traded. It is therefore worthwhile to examine the factor mimicking portfolio returns in the return space. A factor mimicking portfolio is a linear combination of available asset returns, subject to having the same covariance with the test assets with the macroeconomic factors. Factor mimicking portfolios contain the same pricing information as the macroeconomic factors. We construct the factor mimicking portfolios using the Fama-MacBeth approach.

To constract the Fama-MacBeth portfolios, we regress asset returns on inflation factors to obtain their betas. Then, for each quarter, we regress the cross-section of asset returns on the estimated betas. Factor mimicking portfolios are constructed as the time-series of second-step regression coefficients. They have unity exposures to the corresponding factors and are orthogonal to other factors.

[2]For commodity futures, since previous metal consists of gold, platinum, and silver, we only include the precious metal in the estimation.

Panels A through C in Table 6 report the mean, t-statistics, and Sharpe ratios of the factor mimicking portfolios constructed using the Fama-MacBeth method. Columns 1-7 use the cross-section of expanded portfolios in each specific asset class. Column 8 uses the 7 average portfolios and column 9 uses the 35 test portfolios. The average return of core inflation mimicking portfolios are negative and statistically significant. The magnitude of average returns are around -1 percent across all asset classes. The average return of headline inflation mimicking portfolios have different signs for different asset classes and the average return of energy inflation mimicking portfolios are mostly indistinguishable from 0. The mimicking portfolio exercise shows that the financial market prices core inflation consistently in a stable way within and across asset classes.

In the appendix, we report the characteristics of factor mimicking portfolios constructed by an alternative approach: the maximum correlation portfolios. The maximum correlation portfolios for core inflation also exhibit largely consistent average returns.

2.4 Currencies, Commodities, and REITs

2.4.1 Currencies

Currencies and commodity futures are conventionally viewed as inflation-hedging assets, but our previous analysis suggests that they only hedge against energy inflation. It is worth further studying the expanded set of currencies and commodity futures in more detail. In this section, we show that this conclusion applies to all the popular investment strategies that we consider in the expanded set.

The underlying rationale for currencies to hedge inflation risk is the purchasing power parity (PPP). When US experiences a higher inflation, the purchasing power of dollar declines and the foreign currency appreciates. The PPP is a useful benchmark for exchange rates at least in the long run (Rogoff, 1996; Asness et al., 2013; Menkhoff et al., 2017).

Table 7 reports the inflation betas for additional currency portfolios. The value portfolios are sorted on the deviation from PPP (Menkhoff et al., 2017). Portfolio 1 contains currencies that are most undervalued relative to the fundamental currency value, defined as the average real exchange rate between 4.5 and 5.5 years before. Undervalued currencies will revert back to the fundamental values with expected appreciations. Core inflation betas decrease from Portfolio 1 to Portfolio 4. Conditioning on a core inflation shock, currencies that are most undervalued deviate more from their fundamental value based on core inflation. On the contrary, undervalued currencies are expected to appreciate in response to a positive energy inflation shock in the US.

Verdelhan (2018) construct currency portfolios sorted on dollar betas, interacted with the sign of average forward discount. From Portfolio 1 to Portfolio 6, loadings on dollar exchange rate increase. Currencies that have higher dollar betas have more negative core inflation betas and more positive energy inflation betas.

2.4.2 Commodities

Commodity futures are also conventional inflation-hedging assets. As is shown in Table 3, they only hedge against energy inflation. Among commodities, the precious metal, especially gold, is the most well-accepted assets to preserve value. However, this is not true with core inflation. Table 7 shows that gold and platinum have positive core inflation betas that are indistinguishable from zero and they strongly hedge against energy inflation. These precious metal futures have relatively low returns and high volatility.

2.4.3 REITs

A large portion of core inflation is shelter including owners' equivalent rent of residences and rent of primary residence. As of December 2019, the weights of core and shelter in CPI are 79.2% and 33.5%. Therefore, it is natural for investors to consider real estate investment as a good way to hedge against core inflation. Because of the illiquidity of real estate, we focus on REITs.

As is shown in Table 2, REITs behave similar to stocks: it is strongly negatively exposed to core inflation and positively exposed to energy inflation. The two exposures largely offset with each other so it has an insignificant headline inflation beta. Table 7 expands the test assets to REITs in different sectors. The sector portfolios behave quite consistently to the average REITs portfolio in term of headline, core, and energy betas.

Why cannot REITs hedge against core inflation? The correlation of the specific component of shelter inflation and the average REITs return have a very low correlation of -0.07. Unlike the shelter inflation that reflects the change in single-period rents, REITs returns depend on the future rental income and the discount rate. Beause of this difference, REITs is closer to stocks than to shelter inflation.

2.5 Time-varying Exposures and Prices of Risk

In our previous analysis, we examine the case of constant risk exposure and price of risks. Next, we consider how risk exposures and prices of risk vary over time.

A series of studies find that the stock-bond correlation and inflation risk premium changed sign at the turn of the century (Song, 2016; Campbell et al., 2019; Boons et al., 2019). These studies imply a structural break in the dynamic behavior of economic fundamentals and/or changes in the monetary policy regime. Motivated by the potential structural break, we split the sample into two subsamples: the first from 1963 to 1998 and the second from 1999 to 2019. We notice that the relative importance of core and energy inflation changed over time. As we show in Figure 1, core inflation is much more volatile in the early sample and becomes smooth after the mid 1980s. After the late 1990s, energy inflation becomes volatile with large spikes. Therefore, core inflation's contribution to the overall inflation risks decreases in the second subsample, while energy inflation's contribution increases. Because of the change of relative contribution and the opposite signs of stocks' core and energy inflation betas, stocks' exposures to headline inflation may switch signs.

Table 8 reports the inflation exposures in the two sub-samples. The stock's core beta is stable over time, but its energy beta increases from zero before 2000s to significantly positive after 2000s. Overall, the stock's headline beta switches from negative to positive. The bonds' exposures to core are more negative before the 2000s, and their exposures to energy are more negative after the 2000s. Currencies and commodity futures hedge against energy inflation more strongly in the second subsample, and the headline exposures are attributed mostly to energy inflation. REITs have a pattern similar to stocks. Panel C provides the p-value of statistical tests on whether the betas change across the two subsamples. Core betas are stable across asset classes, while headline and energy inflation betas show significant structural changes.

To fully explore the time variation, we use a local least square estimator following Adrian et al. (2015). At any time t, the beta estimate follows

$$[\hat{\alpha}(t), \hat{\beta}(t)']' = arg \min_{(\alpha,\beta)} \sum_{i=1}^{n} \frac{K((t_i - t)/h_k T)}{h_k T} (r^e_{i,t_i} - \alpha - \beta' \varepsilon_{\pi,t_i})^2$$

where $K(z) = 1/\sqrt{2\pi} exp(-z^2/2)$ is a Gaussian density kernel and h_k is a bandwidth. We choose the bandwidth to be 0.05. In Figure 6, the betas slowly evolve over time. While our simple two-regime approach cannot study all the variations, it largely capture the key structural break in a concise way.

As core exposures do not vary much over time, we find that the price of core risks are also stable in both subsamples. Panel D shows that the price of core risks are negative and similar across the two subsamples. The prices of headline risks are insignificant in both samples. Energy inflation has a positive price of risk before the 2000s.

Even though the price of core risk does not show a structural change, it might show medium frequency variation with economic conditions. We specify the stochastic discount factor M_{t+1} and the price of risk λ_t as:

$$\frac{M_{t+1} - E_t M_{t+1}}{E_t M_{t+1}} = -\lambda_t u_{t+1},$$

where $\lambda_t = \Sigma_u^{-\frac{1}{2}}(\lambda_0 + \lambda_1 F_t)$. Under this specification, the expected excess return of asset i can be expressed as $E_t R_{t+1}^i = \beta_i'(\lambda_0 + \lambda_1 F_t)$.

The prices of core and energy inflation risks are both specified as a linear function of economic variable F_t. We choose F_t as the 10-year-3-month term spread and estimate the conditional asset pricing models following the three-step procedure proposed by Adrian et al. (2015). First, we extract the unexpected components of the inflation risk factors. Second, we regress asset returns onto the risk factors as well as lagged values of F_t. Third, λ_0 and λ_1 are constructed using the regression coefficients in the second step. We refer the readers to Adrian et al. (2015) for a detailed description of the method.

We find that the price of core inflation risk decreases with the term spread. We obtain an intercept λ_0 of -0.94 with an t-statistic of -1.79, and a slope λ_1 of -0.52 with an t-statistic of -1.93. The price of energy does not vary with the term spread.

Interestingly, the time-varying core and energy betas can help explain the time varying correlation between stock and bond returns. In our sample, the correlation between the stock market return and the average Treasury return is 0.34 in the first subsample and -0.55 afterwards. Campbell et al. (2019) show that the covariance between inflation and real activity switches signs around the turn of the twenty first century, which affects the changing stock-bond return correlation. Song (2016) investigates the monetary/fiscal policy forces behind this time-varying correlation. Our decomposition of core and energy provides a new economic interpretation of the changing covariation between the real and nominal sides quantities. From the perspective of our inflation decomposition, the nominal-real covariance switches signs because of the changing relative weights of core and energy in headline inflation volatility. In the first subsample, both stocks and bonds expose negatively to core and headline inflation, as core is the dominant force. Thus, stocks and bonds comove. In the second subsample, core inflation is muted while energy emerges as the dominant force, leading to a positive stock exposure to energy and headline inflation. Since bonds still have negative inflation exposure, the stock-bond correlation turns negative.

In sum, core exposure and price of risk are stable over time. There is interesting time variation in energy inflation exposures of different assets, which contributes to changes in their headline inflation exposure. These shifting exposures to inflation risk provide a new perspective on the changing sign of stock-bond return correlation, which has attracted some attention in the macro finance literature.

2.6 Other Macroeconomic Risk Factors

Previous analysis shows that core inflation risks are priced in a wide range of asset classes. Is the core inflation risk simply reflecting information in other known macroeconomic risks? This idea goes back to Fama (1981) who argues that stocks are negatively exposed to inflation because inflation is countercyclical and stock price is procyclical. In this section, we include a set of macroeconomic factors that are suggested in the asset pricing literature and use the set of 35 portfolios for estimation. The macroeconomic factors we consider include the consumption growth rate, durable consumption growth rate (Yogo, 2006), industrial production growth rate (Chen et al., 1986), payroll growth rate, unemployment growth rate, and the long run and short-run consumption growth news constructed by Hansen et al. (2008), unfiltered consumption growth (Kroencke, 2017), and capital share growth (Lettau et al., 2019).

We re-estimate the first-step regression including the macroeconomic factors. The inclusion of macroeconomic factors control for cyclicality and may change the inflation betas and the price of risk estimates. Estimates of price of risks are reported in Table 9. In untabulated results, the first-stage estimates of inflation betas are robust to the macro factor controls. We find that none of these macroeconomic factors can drive out the negative risk premium of core inflation. The price of core inflation risk estimates remain similar both in magnitude and statistical significance across all specifications, and none of the macroeconomic factors is significantly priced in our portfolios across asset classes.

2.7 Price Stickiness and Inflation Cyclicality

What is the economic mechanism that makes core and energy inflation different? In this section, we explore the mechanism by looking at other categorization of inflation risks.

First, core and energy goods and services differ greatly in price stickiness. Energy prices are flexible while core good prices are more sticky. Therefore, we directly look at the inflation of goods and services with flexible and sticky prices. The data are from the Federal Reserve Bank of Atlanta. The sticky price inflation is a weighted basket of sticky-price goods. These goods account for 70% of the whole basket and their prices change relative slowly every 5 to 26 months. The flexible price goods account for 30% and their prices change every 1 to 4 months. The sticky price items are mostly core goods, such as personal care fees, motor vehicle fees, water, sewer, and trash collection services, medical care services, etc. The flexible price items include motor fuel, car and truck rental, fresh fruits and vegetables, etc, which are mostly food and energy goods. The sticky and core inflation share similar properties. Shocks to the core and sticky inflation have a correlation of 0.85, and shocks to the energy and flexible inflation have a correlation of 0.91. When examining inflation risks, we confirm that the asset exposures to sticky and flexible inflation risks and their prices of risk resemble the properties of core and energy inflation. Table 10 reports the results of inflation exposures for 7 average portfolios and the price of risk estimates for both 7 and 35 portfolios. This result indicates that the economic mechanism behind the difference between core and energy inflation could be that they differ in price stickiness.

Second, core good prices comove with the economic conditions differently. We further separate core goods into cyclical and acyclical categories and study their inflation respectively. The data are from the Federal Reserve Bank of San Francisco. Cyclical core items are defined as having a statistically significant negative slope of the Phillips curve, i.e., the regression coefficient of price change on the unemployment gap. The remaining items are acyclical. Table 10 reports the results of inflation exposures for 7 average portfolios and the price of risk estimates for both 7 and 35 portfolios. Despite both being inflation for core goods, only the cyclical component inherits the properties of core inflation, in both β's and the price of risk. That is to say, core inflation exposures and price of risk are mainly from the cyclical component, while the acyclical core has an insignificant price. Stocks and fixed-income securities are bad hedges for the inflation component that reflect news about the macroeconomy, even for core goods, while commodity futures hedge the remaining component. The significantly negative risk premium for cyclical inflation is consistent with our interpretation.[3]

Motivated by these facts, we develop a model with a mechanism of price stickiness. The core goods has higher price stickiness than energy goods and core inflation negatively comoves with economic conditions.

2.8 Extensions

2.8.1 Expected and Unexpected Inflation

In the previous analysis, we examine what assets hedge against unexpected inflation shocks. A practical question for investors is whether an asset can hedge against inflation (Bekaert and Wang, 2010). They often do not distinguish the expected and unexpected inflation. We consider this setting and regress the realized excess return of asset i onto the realized level of inflation (headline, core, and energy) as follows:

$$r^e_{i,t} = \alpha_i + \beta^i_{level}\pi_t + u_{i,t}, \tag{3}$$

[3]We need to note that the sample for this exercise starts from 1988, which is much shorter than the full sample we use in other exercises. Due to the shorter sample and different coverage, the betas are less significant.

where β^i_{level} represents the comovement between excess return and inflation level.

Further, we decompose the level of inflation into the expected and unexpected components, with the expected inflation defined as $E_{t-1}\pi_t = \pi_t - \varepsilon_{\pi,t}$. We run the following regression:

$$r^e_{i,t} = \alpha_i + \beta^i_e \left(E_{t-1}\pi_t\right) + \beta^i_u \varepsilon_{\pi,t} + u_{i,t}. \tag{4}$$

In this specification, β^i_e represents how much asset i's excess return changes with the expected inflation. Since risk-free rate reflects the expected inflation, a zero β^i_e indicates that asset i has the same hedging property against expected inflation as the risk-free rate. If β^i_e lies between -1 and 0, asset i is an imperfect hedge against expected inflation.

Table 11 Panel A reports estimates of equation (3) and (4) for the headline inflation. β_{level} are qualitative similar to β_u, but the magnitude of coefficients for the fixed-income securities and commodity futures are smaller. β_e are not significantly different from zero except for corproate bonds. Therefore, most asset classes can prefectly hedge expected inflation. The results illustrate that hedging expected and unexpected inflation are very different. The overall β_{level} includes both expected and unexpected inflation hedging.

Panel B report the estimates for core and energy inflation. The results of core inflation share the same message as the headline inflation that β_{level} mixes the two types. These assets hedge against the expected core inflation, while it is very difficult to hedge core shocks. We do not include the energy inflation expectation as it is fairly transitory. Exposures to energy inflation, β_{level} are similar with energy shock exposures β_u.

In the appendix, we report the asset pricing test results for the 7 average portfolios with respect to shocks to expected core inflation. The shock to expected core inflation is constructed as $A\varepsilon_{\pi,t}$ and it has a high correlation of 0.90 with the core inflation shock. All results are very close to those with core inflation shocks.

2.8.2 Cash Flow News and Discount Rate News Decomposition

Stock returns can be decomposed into news in expected future cash flows (CF) and discount rates (DR). In the previous section, we show that stock returns load negatively on core inflation and positively on energy inflation. In this section, we further study the exposures of economically distinct CF and DR news components.

We perform standard return decomposition by estimating a VAR(1) and extract CF and DR news (Campbell, 1991). The VAR includes real stock returns, price-dividend ratio, real risk-free rate, and headline inflation. Then, we estimate the risk exposures of CF and DR news to core and energy inflation in a bi-variate regression. The core exposure of CF news is negative with a coefficient of -2.14 (t-stat -4.12), and the core exposure of DR news is positive with a coefficient of 4.23 (t-stat 3.47). Since returns are negatively associated with DR news, both news significantly contributes to the negative core exposure of stock returns. Motivated by this fact, the model that we propose later will feature both CF and DR channels. The exposures of CF and DR news to energy inflation are -0.01 (t-stat -0.23) and -0.19 (t-stat 2.05). Therefore, stocks' energy betas are mainly from the DR news. We obtain similar patterns in an analysis of the cross-section of industry stock portfolios.

3 The Model

In this section, we propose a two-sector New Keynesian model that rationalizes all the empirical findings. Our model is highly stylized, and we consider the model as including the minimum set of ingredients to accommodate our empirical findings.

3.1 Model Setup

3.1.1 Households

Representative households derive utility from a consumption basket C_t, which consists of core goods $C_{c,t}$ and energy goods $C_{e,t}$. The consumption basket is the CES aggregation of the two goods:

$$C_t = \left[\alpha_c C_{c,t}^{\frac{\phi-1}{\phi}} + (1-\alpha_c)(\exp(\delta_t)C_{e,t})^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{5}$$

α_c is the parameter of weight on core consumption. When α_c is larger, households prefer the core good more. ϕ is the elasticity of substitution between core and energy. If ϕ is larger, core and energy goods are more substitutable. δ_t is a shock to the relative demand of energy. Higher δ means relatively higher demand for the energy good.

Households have CRRA utility over the consumption basket and disutility from labor supply. They maximize their lifetime utility subject the budget constraint as follows.

$$\max_{C_{c,t},C_{e,t},B_{t+1},N_t} E\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t^{1-\gamma}-1}{1-\gamma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right]$$

$$s.t.: P_{h,t}P_tC_t + B_{t+1} = W_tN_t + P_{e,t}P_tC_{e,t} + B_t(1+i_t) + \Pi_t$$

The core good is the numeraire and its real price equal 1. $P_{h,t}$ is the real price of the consumption basket (headline price), and $P_{e,t}$ is the real price of energy good. P_t is the nominal price of the core good, W_t is the nominal wage, and Π_t is the profit from firms. B_{t+1} is the amount of nominal risk-free bond held in period t and i_t is the nominal interest rate.

The Euler Euqation is

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{P_{h,t}P_t}{P_{h,t+1}P_{t+1}}(1+i_t)\right]=1$$

The optimal choice between core and energy consumption follows

$$C_{e,t}=\left(\frac{\alpha_c P_{e,t}}{1-\alpha_c}\right)^{-\phi}\exp[(\phi-1)\delta_t]C_{c,t} \tag{6}$$

The optimal consumption labor choice follows

$$W_t C_t^{-\gamma}=P_{h,t}P_t N_t^{\varphi} \tag{7}$$

3.1.2 Core Firms

The core good consists of a continuum of varieties $C_{c,t}(i)$, which are aggregated through a CES aggregator:

$$C_{c,t}=\left(\int_0^1 C_{c,t}(i)^{\frac{\varepsilon_t-1}{\varepsilon_t}}di\right)^{\frac{\varepsilon_t}{\varepsilon_t-1}} \tag{8}$$

ε_t is the elasticity of substitution across varieties. Each variety $i\in[0,1]$ of core goods is produced by a firm in a monopolistic competitive environment. $\exp(\mu_t)\equiv\frac{\varepsilon_t}{\varepsilon_t-1}$ is the desired markup that fluctuates exogenously. This is a modeling device to capture the variation of inflation that are independent of other real macroeconomic and policy shocks. Smets and Wouters (2007) find that markup shocks are the dominant drivers of inflation, while other shocks (e.g. TFP, investment, monetary and fiscal shocks) explain only a minor fraction of inflation. Therefore we abstract away these other shocks in our model.

The production technology of each variety of core good is:

$$C_{c,t}(i)=AN_t(i)^{1-\alpha} \tag{9}$$

A is the constant total factor productivity (TFP) of the economy. The production technology has decreasing returns to scale. In each period, firms face price rigidity. In every period, each firm may adjust its price with probability $1-\theta$. Firms set optimal price facing the demand schedule. They set optimal prices by solving the following maximization problem:

$$\max_{P_t^*}\sum_{k=0}^{\infty}\theta^k E_t M_{t,t+k}^{\$}[P_t^* Y_{t+k|t}-\Psi(Y_{t+k|t})] \tag{10}$$

$$s.t.:Y_{t+k|t}=\left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon_{t+k}}C_{c,t+k} \tag{11}$$

where $Y_{t+k|t}$ is the production if the firm still faces a price P_t^* in period $t+k$, $M_{t,t+k}^{\$}\equiv\beta^k\left(\frac{C_{t+k}}{C_t}\right)^{-\gamma}\frac{P_{h,t}}{P_{h,t+k}}\frac{P_t}{P_{t+k}}$ is the nominal stochastic discount factor from period t to $t+k$, and $\Psi(Y_{t+k|t})$ is the total cost of producing $Y_{t+k|t}$ units of core good. The firm's optimality condition is:

$$\sum_{k=0}^{\infty} \theta^k E_t M^{\$}_{t,t+k} \left[Y_{t+k|t} + (P_t^* - \Psi'(Y_{t+k|t})) \frac{\partial Y_{t+k|t}}{\partial P_t^*} \right] = 0 \tag{12}$$

3.1.3 Energy Endowment

We assume that in each priod, $C_{e,t}$ units of energy good are endowed.

3.1.4 Monetary Policy

The central bank follows the Taylor rule:

$$i_t = \bar{i} + \phi_\pi \pi_t \tag{13}$$

The nominal interest rate responds to current inflation of the core goods. $\phi_\pi > 1$ indicates that when current inflation is high, central bank raises the nominal rate more to fight against the inflation. Through the Taylor rule, asset prices will incorporate future expectations of monetary policy responses to inflation. For simplicity, we omit the response to output and the monetary policy shock.

3.1.5 Exogenous Processes

There are three exogenous processes: the markup, energy endowment, and relative energy demand. The three exogenous shocks all follow first-order autoregressive processes:

$$\mu_t = (1 - \rho_\mu)\bar{\mu} + \rho_\mu \mu_{t-1} + \sigma_\sigma \varepsilon_{\mu,t} \tag{14}$$

$$\log C_{e,t} \equiv c_{e,t} = (1 - \rho_e)\bar{c}_e + \rho_e c_{e,t-1} + \sigma_e \varepsilon_{e,t} \tag{15}$$

$$\delta_t = \rho_\delta \delta_{t-1} + \sigma_\delta \varepsilon_{\delta,t} \tag{16}$$

3.2 Equilibrium Characterization

The model can be solved analytically after we approximate the dynamic economic system with log-linearization. All lower-case letters refer to the log of each variable. With an abuse of notation, all variables should be read as deviations from the deterministic steady state.

We make the following assumption on the parameters of the model.

Assumption 1 *The elasticity of substitution between core and energy is greater than unity, but less than the intertemporal elasticity of substitution, i.e.* $1 < \phi < \frac{1}{\gamma}$.

The following two lemmas express the real price of energy (in core) and the marginal utility of core good.

Lemma 1 *(Energy price) The real energy price is equal to the ratio of core and energy goods, adjusted by the relative demand shock.*

$$p_{e,t} = \left(1 - \frac{1}{\phi}\right)\delta_t + \frac{1}{\phi}(c_{c,t} - c_{e,t}) \tag{17}$$

Lemma shows that energy demand raises energy price while energy supply lowers energy price. Intuitively, when core goods becomes more scarce, energy price increases.

Lemma 2 *(Consumption basket and headline price) Households' consumption basket is a weighted average of core and energy consumption, adjusted by the energy demand shock. The real headline price proportional to real energy price after adjustment of energy demand shock*[4]*:*

$$c_t = \hat{\alpha}_c c_{c,t} + (1 - \hat{\alpha}_c)(c_{e,t} + \delta_t) \tag{18}$$

$$p_{h,t} = (1 - \hat{\alpha}_c)(p_{e,t} - \delta_t) = \frac{1 - \hat{\alpha}_c}{\phi}(c_{c,t} - c_{e,t} - \delta_t) \tag{19}$$

Lemma 2 states that core and energy consumption as well the energy demand shock can effectively raise the consumption basket. For given consumption of core and energy good, the energy demand shock lowers the headline price because less energy is needed to achieve the same level of consumption basket.

Lemma 3 *(Energy effects on the marginal utility of core goods) The marginal utility of core goods is:*

$$MU_{cc,t} = \alpha_c C_t^{\frac{1}{\phi} - \gamma} C_{c,t}^{-\frac{1}{\phi}} \tag{20}$$

For given consumption of the core good, under assumption 3.2, positive shocks on energy supply and demand both increase the marginal utility of core goods.

The marginal utility of core goods is the product of the marginal utility of consumption basket ($C_t^{-\gamma}$) and the partial derivative of consumption basket with respect to core goods ($\alpha_c C_t^{\frac{1}{\phi}} C_{c,t}^{-\frac{1}{\phi}}$). An increased energy supply or demand increase the consumption basket. On one hand, the marginal utility of consumption basket decreases because of higher consumption basket and the magnitude is controlled by intertemporal elasticity of substitution γ. On the other hand, the complementarity between the two goods implies that the marginal utility of core goods increases with energy supply or demand and its magnitude is controlled

[4] $\hat{\alpha}_c = \frac{\alpha_c}{\alpha_c + (1-\alpha_c)\left(\frac{C_e}{C_c}\right)^{\frac{\phi-1}{\phi}}}$ is a constant based on steady-state consumption.

by the elasticity ϕ. Overall, under Assumption , $\frac{1}{\phi} > \gamma$, the marginal utility of core goods increases.

Next, we discuss the two key equations that characterize the equilibrium of the economy.

As in standard New Keynesian models, the core output and inflation can be characterized by the New Keynesian Philips Curve (NKPC) and the dynamic IS (DIS) Curve. The NKPC is written as:

$$\pi_t = \beta E_t \pi_{t+1} - \lambda(-mc_t - \mu_t) \tag{21}$$

where π_t is the inflation of core goods, mc_t is the log real marginal cost of production in the economy, so that $-mc_t$ is the log average markup. μ_t is the desired markup faced by the firms. This equation implies that core inflation depends on future expected core inflation and the deviation of markup from the desired level. λ is a constant that captures the relative weight of future expectation and current markup deviation, with $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\bar{\varepsilon}}$. Inflation increases when the average markup is lower than the desired markup.

Moreover, we can write the average real marginal cost of production as:

$$mc_t = \kappa_c c_{c,t} + \kappa_e c_{e,t} + \kappa_\delta \delta_t \tag{22}$$

where:

$$\kappa_c = \frac{\varphi + \alpha}{1-\alpha} + \gamma\hat{\alpha}_c + \frac{1-\hat{\alpha}_c}{\phi} > 0$$

$$\kappa_e = \kappa_\delta = (\gamma - \frac{1}{\phi})(1-\hat{\alpha}_c) < 0$$

The marginal cost increases with the production of core goods ($\kappa_c > 0$) both because the production technology is decreasing return to scale and the increase in wage when core consumption is higher. A positive energy demand or supply shock reduces marginal cost of production ($\kappa_e < 0$) because it raises the marginal utility of consumption of core goods, which leads to a lower wage level.

The dynamic IS curve is written as:

$$-\eta_c E_t(c_{c.t+1} - c_{c,t}) - \kappa_e[E_t(c_{e,t+1} - c_{e,t}) + E_t(\delta_{t+1} - \delta_t)] - E_t\pi_{t+1} + \phi_\pi \pi_t = 0 \tag{23}$$

where $\eta_c = \gamma\hat{\alpha}_c + \frac{1-\hat{\alpha}_c}{\phi} > 0$.

Combining the NKPC and the DIS curve, we can solve for the core output and inflation.

3.3 Core Output and Inflation

Proposition 1 *(Core output and inflation) The core consumption and core inflation can be expressed as linear combinations of the three state variables: markup, energy supply, and energy demand.*

$$c_{c,t} = c_{c,\mu}\mu_t + c_{c,e}c_{e,t} + c_{c,\delta}\delta_t, \pi_t = \pi_\mu\mu_t + \pi_e c_{e,t} + \pi_\delta\delta_t.$$

The signs of the loadings are given by:

$$c_{c,\mu} < 0, c_{c,e} > 0, c_{c,e} > 0,$$

$$\pi_\mu > 0, \pi_e > 0, \pi_\delta > 0.$$

The formulae for these coefficients are shown in the appendix. The signs of core output and inflation loadings on the markup shock $c_{c,\mu}, \pi_\mu$ are similar with standard New Keynesian models. From the NKPC, a positive markup shock increases the price charged by core firms, so the nominal price of the core good increases. While the aggregate demand curve is not affected by the markup given core output inflation, in equilibrium, core output declines and core inflation rises after a positive markup shock.

The loadings of core output and inflation on the energy supply shock reflect the energy supply's effects on NKPC (aggregate supply) and the aggregate demand of core goods. As in Lemma 2, the increased energy supply increases the marginal utility of core and thus pushes down the wage, the marginal cost of core production. The decreased marginal cost of core production increases the aggregate supply of core goods. On the demand side, the increased the marginal utility of core raises aggregate demand of core goods. Therefore, energy supply shock is expansionary, i.e., $c_{c,e} > 0$. For core inflation, when we compare the slope of aggregate demand curve and NKPC, NKPC has a steeper slope than the aggregate demand curve ($\kappa_c > \eta_c$). Therefore, core inflation increases with energy supply, i.e., $\pi_e > 0$.

We notice from the system of two equations that the energy supply and demand shocks are symmetric in the two equations. Therefore, their effect on the core output and inflation are identical. A positive energy demand shock increases core output and inflation. $c_{c,\delta} > 0, \pi_\delta > 0$.

3.4 Energy Inflation and Headline Inflation

The real price of energy in terms of core goods is

$$p_{e,t} = \frac{1}{\phi}[c_{c,t} - c_{e,t} + (\phi - 1)\delta_t]$$

$$p_{h,t} = \frac{1 - \hat{\alpha}_c}{\phi}(c_{c,t} - c_{e,t} - \delta_t)$$

The nominal energy inflation is the change of real price of energy plus the inflation.

$$\pi_{t+1}+\Delta p_{e,t+1} = -p_{e,t}+\left(\pi_\mu + \frac{1}{\phi}c_{c,\mu}\right)\mu_{t+1}+\left(\pi_e + \frac{1}{\phi}(c_{c,e}-1)\right)c_{e,t}+\left(\pi_\delta + \frac{1}{\phi}(c_{c,\delta}+\phi-1)\right)\delta_t \tag{24}$$

Since $\pi_\mu, c_{c,\mu} < 0$, $\pi_\mu + \frac{1}{\phi}c_{c,\mu} < 0$. A positive markup shock lowers energy inflation.

The energy demand and supply shocks affect energy inflation both directly and indirectly. The direct effect is negative for energy supply shock and positive for energy demand shock, holding core output and inflation fixed. The indirect effects work through core output and inflation: both shocks raise core output and inflation, which in turn translates into higher energy inflation. However, the indirect effects are proportion to the share of energy inflation in the headline inflation $1-\hat{\alpha}_c$. In the data, this share is small, so the direct effects dominate and energy inflation is a result of a negative supply shock or a positive demand shock in energy goods.

The headline inflation is expressed as follows:

$$\pi_{h,t} = \pi_t + \Delta p_{h,t} = \pi_t + \frac{1-\hat{\alpha}_c}{\phi}\Delta(c_{c,t} - c_{e,t} - \delta_t)$$

The exposure of headline inflation to the markup shock is $\pi_\mu + \frac{1-\hat{\alpha}_c}{\phi}c_{c,\mu} < 0$. The sign of headline inflation's loadings on energy supply and demand shocks is ambiguous.

3.5 Event Study: Energy Demand and Supply Shock During the Covid-19 Episode

In this subsection, we calibrate the energy supply and demand shock to match the magnitude of quantity and price drop during the Covid-19 crisis. Parameters are presented in Table B1. Observing the drop in energy consumption quantity and energy price during the Covid-19 episode, we infer the magnitude of energy supply and demand shock. Based on our model, we calculate how much core output drop these two shocks can explain and how much asset prices move in response to these shocks.

In our model, energy inflation is mainly driven by energy price and quantity shocks directly. These two shocks also have indirect effects on core output, but that effect is relatively small compared to the direct effect. Therefore, we ignore the indirect effect.

The period we look at is from 2020 M1 when Covid-19 just started in China and had not affected the world, to 2020 M4 when the world was deeply affected by the pandemic. Energy consumption (in energy units) dropped by 27.0 percent, the energy component of CPI dropped by 17.0 percent. Core consumption dropped by 20.5 percent but the core inflation moved relatively little, by -0.33 percent.

In our model, the energy price is equal to $\frac{1}{\phi}(c_{c,t} - c_{e,t}) + \frac{\phi-1}{\phi}\delta_t = -17.0\%$. We plug in $c_{c,t} = -20.5\%, c_{e,t} = -27.0\%$, we can infer that $\delta_t = -24.0\%$.

The model has an analytical solution after log-linearization. Plug in these energy demand and supply shocks, the core consumption drop is equal to -2.14 percent, about 10% of the total 20 percent drop in core consumption. The core sector in our model is highly stylized — we abstract away the TFP shock and aggregate demand shock in the model, both of which are very important in explaining the large drop in core consumption and mild change in core inflation. This makes it inappropriate to conduct quantitative analysis in our model.

4 Asset Pricing Implications

In this section, we derive the stochastic discount factor (SDF) and the asset prices for core stock, nominal bond, currency, and commodity futures. As core inflation is mainly driven by the markup shock while the effect of energy shocks are small, we interpret the loadings of SDF and asset returns as the loadings on core inflation. For energy inflation shocks, it is a weighted average of loadings of energy demand and supply shock, which we will show in the last subsection.

4.1 Stochastic Discount Factor and the Price of Risk

We consider the SDF in real terms in the unit of consumption basket $M_{t+1} = M^{\$}_{t+1}\frac{P_{t+1}}{P_t}\frac{P_{h,t+1}}{P_{h,t}}$, and denote m_{t+1} be its log term.

Proposition 2 *(The SDF and the price of risk) The stochastic discount factor (in real terms, in the unit of consumption basket) m_{t+1} is expressed as follows:*

$$m_{t+1} = m_\mu(1-\rho_\mu)\mu_t + m_e(1-\rho_e)c_{e,t} + m_\delta(1-\rho_\delta)\delta_t - \lambda_\mu\sigma_\mu\varepsilon_{\mu,t+1} - \lambda_e\sigma_e\varepsilon_{e,t+1} - \lambda_\delta\sigma_\delta\varepsilon_{\delta,t+1}$$

where the signs of the coefficients are are:

$$m_\mu = \gamma\hat{\alpha}_c c_{c,\mu} < 0, m_e = \gamma(\hat{\alpha}_c c_{c,e} + 1 - \hat{\alpha}_c) > 0,$$

$$m_\delta = \gamma(\hat{\alpha}_c c_{c,e} + 1 - \hat{\alpha}_c) > 0,$$

$$\lambda_\mu = \gamma\hat{\alpha}_c c_{c,\mu} < 0, \lambda_e = \gamma(\hat{\alpha}_c c_{c,e} + 1 - \hat{\alpha}_c) > 0, \lambda_\delta = \gamma(\hat{\alpha}_c c_{c,\delta} + 1 - \hat{\alpha}_c) > 0.$$

When there is a positive markup shock, core consumption decreases and the price of risk λ_μ is negative.

For either energy demand or supply, a positive shock increases the consumption basket so that the marginal utility of the consumption basket is lower. While the energy demand and supply shock has the same price of risk, their effects on energy prices are different. This gives us an explanation of why the price of energy inflation is not significant in the data. The sign of the price of energy inflation risk depends on whether it is energy demand or supply that mainly drives the energy price.

Conditional on the markup, the price of energy is driven by both the energy supply and demand shock. While both shocks increases the marginal utility of core good, they have opposite effects on the energy price. Ignoring the indirect effect of energy shocks on energy price through core consumption and core inflation, we obtain the following expression for the price of energy inflation risk:

$$\lambda_{energy} \propto -\frac{1}{\phi}\lambda_e\sigma_e^2 + \frac{\phi-1}{\phi}\lambda_\delta\sigma_\delta^2$$

There are two offsetting forces determining the price of energy inflation risks, one from energy supply $-\frac{1}{\phi}\lambda_e\sigma_e^2$, and the other from energy demand $\frac{\phi-1}{\phi}\lambda_\delta\sigma_\delta^2$. If the energy demand dominates energy supply in driving the energy inflation, the price of energy inflation is positive. Otherwise, the price of energy inflation is negative. The offsetting forces helps us understand why it is hard to identify a significant price of energy inflation risk from the test portfolios.

When we discuss asset returns in some asset classes, especially bonds, we need to use the nominal SDF. The nominal SDF $m_{t+1}^{\$}$ is expressed as $m_{t+1}^{\$} = m_{t+1} - (\pi_{t+1} + \Delta p_{h,t+1})$, where $\pi_{t+1} + \Delta p_{h,t+1}$ is the headline inflation. We see that if inflation is neutral and does not affect the real SDF, the nominal price of inflation (and all its components) should be 1. Any deviation is caused by the effect of shocks that drive inflation (both core and energy) on the real economic quantities.

4.2 Core Stock Returns

Having solved for the SDF, we move forward to solve for asset prices in this economy, starting with the stock returns. A core stock is a claim to the core output (net of labor cost). Our first step is to express the dividend as a log-linear function of the three exogenous state variables.

The dividend is equal to core output net of labor cost, and we express the dividend in the unit of headline price.

$$D_t = \frac{1}{P_{h,t}}(C_{c,t} - \frac{W_t}{P_t}N_t)$$

We make the following assumption on the steady state level of the markup.

Assumption 2 *The steady state level of the markup $\bar{\mu}$ is sufficiently large, i.e.,*

$$\exp(-\bar{\mu}) < \frac{1}{(1-\alpha)(\gamma + \frac{\varphi+1}{1-\alpha})}.$$

At the steady state, labor income accounts for $(1-\alpha)\exp(-\bar{\mu})$ share of the total output, so that dividend accounts for the remaining $1-(1-\alpha)\exp(-\bar{\mu})$ share. Log-linearize this equation around the steady state:

$$d_t = \frac{1}{1-(1-\alpha)\exp(-\bar{\mu})}[c_{c,t} - (1-\alpha)(w_t - p_t + n_t)] - p_{h,t}$$

Lemma 4 *Under Assumption 2, when the average level of markup $\bar{\mu}$ is sufficiently large, $d_\mu < 0, d_e > 0, d_\delta > 0$.*

Postulate the log price-dividend ratio as $z_t = z_\mu \mu_t + z_e c_{e,t} + z_\delta \delta_t$. Denote $r_{s,t+1}$ the return to the core stock in period $t+1$, according to Campbell-Shiller decomposition

$$r_{s,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta d_{t+1} \tag{25}$$

The coefficients z_μ, z_e, z_δ can be solved from the Euler equation $E_t(m_{t+1} + r_{s,t+1}) + \frac{1}{2}var_t(m_{t+1} + r_{s,t+1}) = 0$. Then we plut z_{t+1} into equation (25) and we arrive at the following proposition.

Proposition 3 *(Core stock return) The real core stock return $r_{s,t+1}$ is expressed as an affine linear combination of the three state variables:*

$$r_{s,t+1} = r_{s0} + r_{s,\mu}\mu_{t+1} + r_{s,e}c_{e,t+1} + r_{s,\delta}\delta_{t+1}. \tag{26}$$

We can determine the signs of the coefficients as follows:

$$r_{s,\mu} < 0, r_{s,e} > 0, r_{s,\delta} > 0.$$

Core stock returns decrease with positive markup shock, negative energy supply and demand shock.

The loadings of core stock returns on the three shocks are intuitive. Core stock return declines with a positive markup because the markup shock is contractionary. For energy demand and supply shocks, both of them are expantionary, so core stock returns increase with positive shocks on energy supply and demand. As in the previous section, we can write the loadings of core stock returns on energy inflation as

$$\beta_{s,energy} \propto -\frac{1}{\phi} r_{s,e}\sigma_e^2 + \frac{\phi-1}{\phi} r_{s,\delta}\sigma_\delta^2$$

Again, the sign of the loadings of core stock return on energy inflation depends on whether supply or demand shock is dominant in determining the energy inflation. From our empirical evidence of $\beta_{s,energy} > 0$, energy demand shocks are the main drivers of energy prices.

4.3 Bond Returns

In this section, we calculate the nominal return to long-term bonds. We focus our discussion in default-free Treasuries and do not discuss credit risk. To calculate the long-term nominal bond returns, we need to use the nominal stochastic discount factor $M_{t+1}^{\$}$. Here, we consider a two-period bond for analytical solution. For bonds with longer maturities, we can follow the same steps and obtain a recursive solution. Denote $P_t^{(1)}$ the price of one-period bond at time t, and $P_t^{(2)}$the price of two-period bond at time t. The following Euler equations holds:

$$E_t M_{t+1}^{\$} P_{t+1}^{(1)} = P_t^{(2)}, E_t M_{t+1}^{\$} P_t^{(1)} = 1.$$

Proposition 4 *Denote $r_{b,t+1}$ be the $t+1$-realized holding-period return of a two-period long-term bond issued at time t, it is an affine linear combination of the three state variables:*

$$r_{b,t+1} = r_{b,0} + r_{b,\mu}\mu_{t+1} + r_{b,e}c_{e,t+1} + r_{b,\delta}\delta_{t+1} \tag{27}$$

The loadings and their signs are as follows:

$$r_{b,\mu} < 0, r_{b,e} < 0, r_{b,\delta} < 0$$

Nominal bond returns decrease with higher markup, higher energy supply, and higher energy demand.

The exposure of long-term bond return to all three shocks are negative. When there is a positive markup shock, consumption goes down and bond price decreases. Moreover, the contemporaneous inflation makes nominal bond less attractive further and the bond price to drop.

The positive energy shock (either demand or supply) has two effects on the nominal bond return. First, it raises the consumption basket. This effect increases the bond price. Second, it raises the inflation expectations and bond price decreases. As shown in the proof in the appendix, the first and third effects dominate the second, so that nominal bond returns decrease with a positive energy shock.

As in our analysis for the SDF and core stocks, bond returns' energy inflation beta is a weighted average of their betas on energy supply and demand shocks. When energy demand shocks are dominant, nominal bond returns comove negatively with energy price. In the data, nominal bonds are indeed negatively exposed to energy inflation, so the bond market evidence reaffirms that energy demand plays a dominant role in driving energy prices.

4.4 Currency Returns

Now, we consider the foreign currency returns. Assume the financial market is complete, the real exchange rate is equal to $\Delta q_{t+1} = m^*_{t+1} - m_{t+1}$. Holding the foreign SDF fixed, foreign exchange rate moves in the opposite direction to m_{t+1}. Therefore we have the following proposition:

Proposition 5 *(Foreign currency returns) Denote $r_{fx,t+1}$ the nominal return to a long position in a foreign currency that is realized at time $t+1$. We can again write it as a linear function of the three state variables:*

$$r_{fx,t+1} = \lambda^{\$}_{\mu}\mu_{t+1} + \lambda^{\$}_{e}c_{e,t+1} + \lambda^{\$}_{\delta}\delta_{t+1}.$$

where $\lambda^{\$}_{\mu} < 0$, and the sign of $\lambda^{\$}_{e}$ and $\lambda^{\$}_{\delta}$ are ambiguous, depending on the relative strength of three forces: the change of real SDF m, the change of headline relative price p_h, and the inflation rate of core goods π.

A positive supply shock increases the real SDF and depreciates the foreign currency, while a nominal inflation appreciates the foreign currency. Combine the two effects, the real exchange rate effect dominates the nominal effect so that the foreign currency loads negaitively on the markup shock. A positive energy shock has multiple effects on the foreign currencies. First, the real SDF decreases ($\lambda_e > 0$) and the real foreign exchange rate appreciates. Second, a positive energy shock is associated with a lower relative headline price p_h. Third, inflation rate of core goods increases with the energy shock $\pi_e > 0$. We cannot decide the sign of changes in nominal price of headline goods $\pi_t + \Delta p_{h,t}$, so that the loading of nominal exchange rate is ambiguous and depends on the parameters. Particularly, the magnitude of relative headline price change depends on the substitutability between goods ϕ. If ϕ is relatively large and the two goods are more substitutable, the change in relative price will be smaller and the foreign currency's loading on energy shocks are more positive.

4.5 Commodity Future Returns

Finally, we derive the price of commodity futures and its exposure to the three fundamental shocks. Denote F_t the price of commodity future, which satisfies the Euler equation:

$$E_t M_{t+1} \frac{P_{e,t+1}}{P_{h,t+1}} = E_t M_{t+1} F_t$$

Proposition 6 *(Commodity future returns) The price of commodity futures is expressed as:*

$$f_t = f_0 + f_\mu \mu_t + f_e c_{e,t} + f_\delta \delta_t \tag{28}$$

where:

$$f_\mu < 0, f_e < 0, f_\delta > 0.$$

Proposition 6 is very intuitive. Under both energy supply and energy demand shock, commodity future returns move in the same direction as the energy inflation so that commodity futures have unambiguously positive loadings on energy inflation risk. The model also implies that commodity future has negative exposure to the markup shock, because higher markup reduces core production and makes core good more scarce. In the data, the exposure of commodities to core inflation is very poorly estimated and it is hard for us to detect this feature with noisy data.

4.6 Extension: Heterogeneous Agents

One limitation of the model is the implication of the consumption-based CAPM. Inflation is not an additional risk factor if a well-measured aggregate consumption growth is included. However, in the empirical section, we find a limited role of aggregate consumption and a significant core inflation risk premium.

Besides the consumption measurement issue, we discuss a theoretical possibility that inflation affect agents in a heterogeneous way. In the online appendix, we extend our model by introducing two agents: workers and shareholders. Workers supply labor and only trade goods in the spot market. They have no access to the financial market. Shareholders hold and price financial assets. As a result, inflation affects asset prices through shareholders' consumption but is disconnected from the aggregate consumption. We show that the propositions on stochastic discount factors and asset risk exposures are preserved in the extended model.

5 Conclusion

In this paper, we study the inflation hedging properties of different asset classes and the price of inflation risks in the asset market. We decompose the headline inflation into core, food, and energy and show they have distinct volatilities and persistence. While stocks exposures to headline inflation are insignificant and currencies and commodities hedge headline inflation, things look different after our decomposition. Conventional real assets of stocks, currencies, and commodity futures only hedge against the energy inflation. We estimate the price of core and energy risks both within and across asset classes. We find that only core inflation carries a negative premium and the magnitude is consistently estimated across all asset classes. This finding implies that hedging against core inflation is costly while the cost of hedging against energy inflation is indistinguishable from zero. We develop a two-sector New Keynesian model to accommodate our empirical findings. Through the lens of our model, the evidence from asset prices suggests that energy demand is the dominant driver of energy prices.

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Table 1: Summary Statistics of Inflation

A. Summary

	Mean	S.D.	Autocorr
Headline	3.76	3.24	0.60
Core	3.75	2.66	0.79
Food	3.75	4.04	0.43
Energy	4.01	19.52	0.04

B. Regression

	β	$s.e.$
Core	0.71	0.01
Food	0.20	0.01
Energy	0.09	0.00

C. Correlation

	Headline	Core	Food	Energy
Headline	1.00			
Core	0.80	1.00		
Food	0.60	0.44	1.00	
Energy	0.69	0.20	0.17	1.00

Notes: This table provides summary statistics of the headline inflation and its three components, core, food, and energy inflation. Data are quarterly from 1963Q3 to 2019Q4. All numbers are annualized. Panel A reports the mean, standard deviation, and autocorrelation of each series. Panel B reports the regression coefficients of headline inflation on core, food, and energy inflation. Panel C reports the correlation matrix.

Table 2: Average Portfolio Exposures to Inflation Risks

			A. Headline		B. Core and Energy			
	Mean	S.D.	headline β	t-stat	core β	t-stat	energy β	t-stat
Stock	6.80	16.79	-1.33	(-1.38)	-5.60	(-3.69)	0.21	(1.81)
Treasury	2.07	6.90	-2.53	(-7.06)	-2.51	(-4.27)	-0.20	(-4.57)
Agency	2.44	5.10	-1.62	(-5.42)	-2.25	(-4.28)	-0.09	(-2.75)
Corporate	3.08	6.39	-1.60	(-4.38)	-2.98	(-4.91)	-0.05	(-1.08)
Currency	1.76	7.05	1.04	(2.02)	-1.04	(-0.65)	0.13	(2.54)
Commodity	4.47	21.90	8.59	(7.53)	-0.07	(-0.04)	1.10	(8.21)
REIT	7.96	17.46	0.31	(0.27)	-6.54	(-3.30)	0.31	(2.48)

Notes: This table reports the mean and standard deviation of the portfolios returns. It also reports the regression results of the specification $r^e_{i,t} = \alpha_i + \beta^i_\pi \varepsilon_{\pi,t} + u_{i,t}$ for 7 average portfolios in each asset class. Panel A uses headline inflation shock as the risk factor. Panel B uses core and energy inflation jointly as risk factors. The t-statistics are in the parenthese.

Table 3: Asset Return Exposure to Inflation Risks

	Mean	S.D.	A. Headline		B. Core and energy			
			Headline β	t-stat	core β	t-stat	energy β	t-stat
				Stock				
Consumer	7.83	17.70	-2.62	(-2.61)	-6.34	(-3.97)	0.06	(0.48)
Manufacturing	6.65	15.49	0.32	(0.35)	-4.20	(-3.02)	0.36	(3.39)
High Tech	7.31	20.29	-1.17	(-1.00)	-6.07	(-3.29)	0.26	(1.86)
Health	8.67	17.80	-2.73	(-2.70)	-6.30	(-3.91)	0.04	(0.34)
Others	7.27	20.09	-2.38	(-2.08)	-7.40	(-4.09)	0.17	(1.22)
				Treasury				
1-year	0.96	1.87	-0.56	(-5.60)	-0.84	(-5.20)	-0.03	(-2.20)
3-year	1.19	3.16	-0.97	(-5.70)	-1.44	(-5.26)	-0.05	(-2.24)
5-year	1.93	5.86	-1.85	(-5.90)	-2.21	(-4.34)	-0.13	(-3.28)
7-year	2.35	6.97	-2.33	(-6.31)	-2.46	(-4.08)	-0.18	(-3.89)
10-year	2.19	8.29	-2.68	(-6.07)	-3.10	(-4.30)	-0.19	(-3.40)
20-year	2.95	11.32	-4.16	(-7.05)	-3.79	(-3.92)	-0.35	(-4.82)
30-year	2.94	13.29	-5.18	(-7.60)	-3.72	(-3.33)	-0.51	(-6.00)
				Agency Bond				
1-5 year	1.83	3.94	-1.17	(-4.99)	-1.90	(-4.66)	-0.05	(-2.03)
5-10 year	3.58	5.20	-1.48	(-3.89)	-0.26	(-0.21)	-0.14	(-3.70)
10-15 year	3.62	8.64	-2.84	(-5.69)	-3.71	(-4.25)	-0.18	(-3.10)
>15 year	4.76	10.38	-3.42	(-5.72)	-3.63	(-3.44)	-0.26	(-3.66)
				Corporate Bond				
1-3 year	2.26	3.21	-0.48	(-2.44)	-1.56	(-4.69)	0.02	(0.70)
3-5 year	2.93	4.89	-0.84	(-2.78)	-2.14	(-4.17)	0.00	(0.06)
5-10 year	3.61	6.91	-1.25	(-2.93)	-2.98	(-4.05)	-0.01	(-0.26)
>15 year	4.27	10.13	-2.85	(-4.98)	-4.47	(-4.66)	-0.13	(-1.91)
				Currency				
Dollar-carry	5.34	8.82	-0.98	(-1.52)	-4.17	(-2.08)	0.00	(-0.04)
Carry-1	-1.81	7.94	0.33	(0.57)	-0.52	(-0.28)	0.06	(0.95)
Carry-2	-0.25	7.47	1.60	(2.99)	1.72	(1.03)	0.14	(2.55)
Carry-3	1.12	7.27	1.02	(1.92)	-0.04	(-0.02)	0.11	(2.02)
Carry-4	2.53	8.20	0.45	(0.74)	-2.50	(-1.34)	0.10	(1.60)
Carry-5	3.43	8.76	1.44	(2.28)	-1.28	(-0.65)	0.19	(2.94)
Carry-6	5.56	10.10	1.38	(1.87)	-3.62	(-1.60)	0.20	(2.72)
				Commodity				
Livestock	2.70	16.99	1.24	(1.24)	-1.09	(-0.66)	0.15	(1.22)
Industrial metal	4.23	25.69	4.73	(2.98)	-1.07	(-0.39)	0.66	(3.66)
Precious metal	3.41	20.96	3.28	(2.65)	-0.22	(-0.11)	0.43	(2.96)
Energy	7.26	36.93	16.51	(7.05)	-0.76	(-0.11)	1.78	(7.54)
Agriculture	0.28	22.24	4.20	(3.28)	2.06	(0.96)	0.26	(1.66)
				REIT				
Equity	8.31	17.87	0.72	(0.61)	-6.48	(-3.20)	0.35	(2.77)
Mortgage	4.73	21.15	-2.25	(-1.63)	-8.61	(-3.56)	0.04	(0.25)
Hybrid	8.20	20.31	-1.05	(-0.79)	-6.14	(-2.60)	0.12	(0.79)

Note: This table reports the mean and standard deviation of the portfolios returns. It also reports the regression results of the specification $r^e_{i,t} = \alpha_i + \beta^i_\pi \varepsilon_{\pi,t} + u_{i,t}$ for 35 test portfolios in each asset class. Panel A uses headline inflation shock as the risk factor. Panel B uses core and energy inflation jointly as risk factors. The t-statistics are in the parenthese.

Table 4: Price of Inflation Risks

	A. 7 Average Portfolios		B. 35 Portfolios	
headline λ	0.22		-0.03	
t-stat	(0.75)		(-0.13)	
core λ		-1.07		-1.06
t-stat		(-3.14)		(-3.91)
energy λ		3.88		3.80
t-stat		(1.35)		(1.37)
R^2	0.44	0.99	0.41	0.80

Notes: This table reports the price of risk estimated from the test portfolios. Panel A uses the 7 average portfolios from each asset class as test portfolios. Panel B uses the 35 test portfolios as test assets. In each panel, the first column reports the price of headline inflation and the second column reports the price of core and energy inflation. Price of risk is estimated using two-step procedure and the t-statistics are calculated using Shanken-adjusted standard errors. The second-step R^2 is also reported in the last row.

Table 5: Price of Inflation Risks

	Stock	Treasury	Agency	Corporate	Currency	Commodity	REIT	Average	All
core λ	-1.26	-0.89	-0.68	-1.09	-1.01	-0.80	-1.06	-1.07	-1.06
t-stat	(-2.51)	(-2.43)	(-1.57)	(-2.75)	(-1.98)	(-0.75)	(-2.70)	(-3.14)	(-3.91)
energy λ	2.02	0.56	-8.25	7.65	2.64	4.18	3.27	3.88	3.80
t-stat	(0.50)	(0.14)	(-1.06)	(2.01)	(0.29)	(1.41)	(0.41)	(1.35)	(1.37)
R^2	0.26	0.93	0.96	0.75	0.69	0.89	0.23	0.99	0.80

Notes: This table reports the price of risks in various specifications. Columns 1 to 7 use a cross-section of expanded portfolios from each asset class to estimate the price of core and energy risk. Column "Average" uses 7 average portfolios and column "All" uses 35 test portfolios. The price of risk is estimated using two-step procedure and the t-statistics are calculated using Shanken-adjusted standard errors. The second-step R^2 is also reported in the last row.

Table 6: Mimicking Portfolios: Fama-MacBeth Portfolios

	Stock	Treasury	Agen	Corp	Curr	Comm	REIT	Aver	All
				A. Core					
mean	-1.26	-0.86	-0.68	-1.05	-1.15	-1.38	-1.05	-0.94	-0.98
t-stat	(-3.31)	(-2.84)	(-2.09)	(-3.06)	(-3.98)	(-1.16)	(-3.25)	(-3.05)	(-3.58)
Sharpe ratio	-0.44	-0.36	-0.27	-0.49	-0.65	-0.17	-0.51	-0.42	-0.49
$corr(r_{fmp},\pi)$	0.25	0.30	0.28	0.35	0.23	0.12	0.36	0.32	0.36
				B. Energy					
mean	2.02	0.64	-8.25	6.66	1.62	12.73	3.47	5.25	5.60
t-stat	(0.61)	(0.19)	(-1.30)	(2.07)	(0.22)	(1.88)	(0.55)	(2.04)	(2.06)
Sharpe ratio	0.09	0.03	-0.18	0.30	0.04	0.36	0.09	0.28	0.28
$corr(r_{fmp},\pi)$	0.41	0.37	0.24	0.44	0.30	0.34	0.28	0.50	0.49
				C. Headline					
mean	-2.81	-0.80	-1.39	-1.40	0.82	1.07	0.89	0.21	-0.04
t-stat	(-3.36)	(-2.24)	(-3.07)	(-2.85)	(0.96)	(1.61)	(1.12)	(0.66)	(-0.13)
Sharpe ratio	-0.45	-0.30	-0.46	-0.42	0.19	0.29	0.18	0.09	-0.02
$corr(r_{fmp},\pi)$	0.18	0.43	0.40	0.37	0.25	0.34	0.24	0.51	0.55

Notes: The table reports the characteristics of Fama-MacBeth factor mimicking portfolios. The table reports the mean, the t-statistics, the Sharpe ratio, and the correlation between the portfolio and the corresponding inflation factor. The columns indicate the test assets used to construct the mimicking portfolios.

Table 7: Inflation Hedging Properties of Currencies, Commodity Futures, and REITs

			A. Headline		B.Core and energy			
	Mean	S.D.	β	t-stat	β	t-stat	β	t-stat
			Currency					
Value-1	0.65	9.90	0.84	(1.15)	-0.93	(-0.41)	0.13	(1.68)
Value-2	0.71	9.74	1.90	(2.72)	-1.72	(-0.79)	0.23	(3.25)
Value-3	1.72	9.58	1.75	(2.53)	-2.72	(-1.28)	0.23	(3.33)
Value-4	4.60	9.17	1.79	(2.72)	-2.93	(-1.45)	0.24	(3.66)
Dollar-beta-1	0.83	3.80	-0.37	(-1.24)	-0.04	(-0.04)	-0.04	(-1.39)
Dollar-beta-2	1.68	5.61	-0.82	(-1.90)	-1.46	(-1.04)	-0.05	(-1.20)
Dollar-beta-3	2.57	6.93	-0.30	(-0.56)	-1.77	(-1.01)	0.02	(0.34)
Dollar-beta-4	3.65	8.16	0.57	(0.90)	-3.27	(-1.61)	0.12	(1.99)
Dollar-beta-5	3.13	10.03	-0.79	(-1.02)	-3.85	(-1.52)	0.01	(0.07)
Dollar-beta-6	4.87	10.59	-0.62	(-0.75)	-5.05	(-1.91)	0.04	(0.46)
			Commodity					
Gold	1.98	17.28	2.14	(1.97)	1.74	(0.91)	0.24	(1.92)
Silver	3.52	31.82	4.95	(2.63)	-0.09	(-0.03)	0.68	(3.06)
Platinum	4.36	20.46	3.40	(2.29)	7.51	(1.63)	0.26	(1.69)
			REIT					
Diversified	7.80	20.89	-0.20	(-0.14)	-7.25	(-3.02)	0.27	(1.82)
Healthcare	11.63	19.18	-0.07	(-0.05)	-7.18	(-1.63)	0.09	(0.61)
Industrial/office	6.84	22.08	2.08	(1.44)	-4.90	(-1.93)	0.43	(2.73)
Lodging/resorts	3.62	32.08	1.20	(0.57)	-5.26	(-1.40)	0.38	(1.61)
Residential	9.91	19.65	-0.65	(-0.51)	-9.63	(-4.40)	0.30	(2.18)
Retail	9.21	19.58	1.54	(1.20)	-4.72	(-2.09)	0.38	(2.68)
Self-storage	10.97	20.67	-0.08	(-0.05)	-7.19	(-1.57)	0.06	(0.36)
Unclassified	7.36	19.05	0.55	(0.44)	-5.86	(-2.68)	0.31	(2.24)

Notes: This table reports the mean and standard deviation of the portfolios returns. It also reports the regression results of the specification $r^e_{i,t} = \alpha_i + \beta^i_\pi \varepsilon_{\pi,t} + u_{i,t}$. Panel A uses headline inflation shock as the risk factor. Panel B uses core and energy inflation jointly as risk factors. The t-statistics are in the parenthese.

Table 8: Subsample Analysis

	A. Headline		B. Core and Energy				C. Test Break p-value		
	headline	t-stat	core	t-stat	energy	t-stat	headline	core	energy
		1963-1998							
Stock	-5.26	(-4.13)	-5.22	(-3.36)	-0.18	(-0.74)			
Treasury	-2.96	(-5.57)	-2.86	(-4.40)	-0.20	(-1.94)			
Agency	-2.59	(-4.26)	-2.58	(-3.53)	-0.12	(-1.12)			
Corporate	-3.33	(-5.70)	-3.14	(-4.38)	-0.23	(-2.20)			
Currency	-0.09	(-0.06)	-0.59	(-0.19)	0.08	(0.42)			
Commodity	4.15	(2.53)	0.68	(0.34)	0.47	(1.55)			
REIT	-6.22	(-4.09)	-5.90	(-3.38)	-0.52	(-1.94)			
		1999-2019							
Stock	2.53	(1.83)	-7.16	(-1.35)	0.32	(2.32)	0.00	0.72	0.04
Treasury	-2.14	(-4.57)	0.56	(0.31)	-0.22	(-4.77)	0.27	0.10	1.19
Agency	-1.01	(-4.18)	0.16	(0.17)	-0.10	(-4.20)	0.01	0.08	0.88
Corporate	-0.19	(-0.48)	-0.35	(-0.23)	-0.01	(-0.33)	0.00	0.15	0.05
Currency	1.30	(2.75)	-1.18	(-0.65)	0.14	(2.99)	0.32	0.74	0.57
Commodity	12.51	(8.47)	-2.03	(-0.36)	1.27	(8.83)	0.00	0.67	0.05
REIT	3.62	(2.37)	-8.29	(-1.42)	0.44	(2.94)	0.00	0.71	0.02

D. Price of Risk

	1963-1998		1999-2019	
headline λ	-0.17		0.28	
t-stat	(-0.61)		(0.65)	
core λ		-1.01		-0.75
t-stat		(-3.47)		(-2.05)
energy λ		4.50		-0.34
t-stat		(1.95)		(-0.07)
R^2	0.37	0.42	0.34	0.49

Notes: This table reports the regression results of the specification $r_{i,t}^e = \alpha_i + \beta_i \varepsilon_{\pi,t} + u_{i,t}$ for 7 average portfolios in each asset class. Panel A uses headline inflation shock as the risk factor. Panel B uses core and energy inflation jointly as risk factors. The t-statistics are in the parenthese. The upper panels show the results from 1963-1998 and the lower panels show the results from 1999-2019. Panel C shows the p-value of tests on equal β over the two sub-sample. Panel D reports the price of risk estimated from the test portfolios. Price of risk is estimated using two-step procedure and the t-statistics are calculated using Shanken-adjusted standard errors. The second-step R^2 is also reported in the last row.

Table 9: Price of Inflation Risks with Macroeconomic Factors

	Cons	Cons/Dur	IP	Pay	Unem	HHL	Unf Cons	Cap
core	-1.08	-1.06	-1.06	-1.04	-1.05	-1.06	-1.08	-1.06
t-stat	(-3.64)	(-3.63)	(-3.65)	(-2.98)	(-3.30)	(-3.42)	(-3.89)	(-3.96)
energy	4.01	4.81	4.21	3.73	3.89	4.09	3.95	4.04
t-stat	(1.26)	(1.35)	(1.41)	(1.35)	(1.37)	(1.27)	(1.38)	(1.46)
macro	0.06	0.09	-0.47	-0.17	0.15	0.41	-0.36	0.00
t-stat	(0.11)	(0.15)	(-0.34)	(-0.26)	(0.35)	(0.50)	(-0.65)	(0.09)
macro2		-3.32				-0.01		
t-stat		(-0.77)				(-0.47)		
R^2	0.80	0.81	0.81	0.80	0.80	0.80	0.79	0.81

Notes: This table reports the price of risks in various specifications with the inclusion of macroeconomic factors including consumption growth (Cons), durable consumption growth (Cons/Dur), industrial production growth (IP), payroll growth (Pay), unemployment rate growth(Unem), long-run consumption growth rate and short-run consumption growth news (HHL), unfiltered consumption growth (Unf Cons), and capital share growth (Cap).

Table 10: Flexible vs. Sticky and Cyclical vs. Acyclical Inflation

	sticky	t-stat	flexible	t-stat	cyclical	t-stat	acyclical	t-stat
A. Asset Return Exposures								
Stock	-4.68	(-2.99)	0.25	(0.61)	-34.45	(-2.07)	-5.98	(-0.60)
Treasury	-1.12	(-1.86)	-0.93	(-5.93)	-5.19	(-0.78)	-3.21	(-0.80)
Agency	-0.94	(-1.93)	-0.51	(-4.20)	-4.93	(-1.28)	-2.08	(-0.89)
Corporate	-1.61	(-2.70)	-0.39	(-2.56)	-10.34	(-2.10)	0.08	(0.03)
Currency	-1.14	(-0.69)	0.41	(2.16)	-12.56	(-1.84)	2.82	(0.68)
Commodity	-1.53	(-0.87)	3.88	(8.51)	-0.40	(-0.02)	49.62	(3.35)
REIT	-4.35	(-2.38)	0.61	(1.38)	-30.75	(-1.63)	-1.85	(-0.16)
B. Price of Risks								
7 portfolios	-1.66	(-2.73)	0.39	(0.39)	-0.26	(-1.92)	0.04	(0.29)
35 portfolios	-1.47	(-3.74)	-0.22	(-0.25)	-0.23	(-2.37)	0.07	(0.51)

Notes: This table reports the two-step asset pricing results for flexible and sticky inflation (left) and cyclical and acyclical inflation (right). Panel A reports the asset return exposures for the 7 average portfolios, and Panel B reports the price of risk estimates for both 7 portfolios and 35 portfolios. The t-statistics are in the parenthese.

Table 11: Hedging Inflation, Expected Inflation, and Inflation Shocks

A. Headline

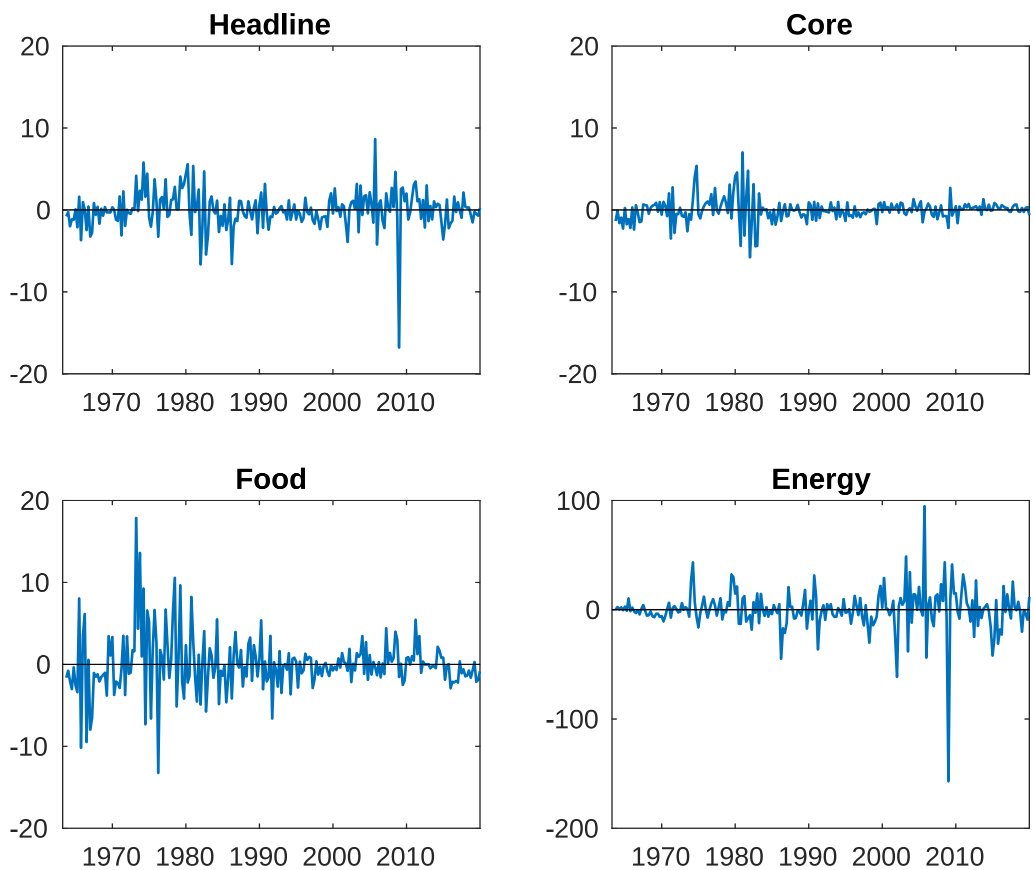
	headline	t-stat	headline expect.	t-stat	headline shock	t-stat
Stock	-1.35	(-1.96)	-1.37	(-1.38)	-1.33	(-1.38)
Treasury	-1.44	(-5.38)	-0.29	(-0.77)	-2.53	(-7.05)
Agency	-1.02	(-4.47)	-0.30	(-0.92)	-1.62	(-5.44)
Corporate	-1.18	(-4.58)	-0.75	(-2.04)	-1.59	(-4.38)
Currency	0.89	(1.74)	0.00	(0.00)	1.04	(1.95)
Commodity	4.50	(5.20)	0.20	(0.17)	8.59	(7.51)
REIT	-0.04	(-0.05)	-0.50	(-0.36)	0.25	(0.21)

B.Core and Energy

	core	t-stat	energy	t-stat	core expect.	t-stat	core shock	t-stat	energy shock	t-stat
Stock	-2.43	(-2.87)	0.16	(1.36)	-0.91	(-0.93)	-5.60	(-3.69)	0.21	(1.81)
Treasury	-0.68	(-2.03)	-0.20	(-4.48)	-0.20	(-0.54)	-2.51	(-4.26)	-0.20	(-4.56)
Agency	-0.69	(-2.35)	-0.10	(-2.93)	-0.27	(-0.86)	-2.28	(-4.32)	-0.09	(-2.74)
Corporate	-1.16	(-3.52)	-0.06	(-1.41)	-0.58	(-1.58)	-2.99	(-4.95)	-0.05	(-1.06)
Currency	-0.19	(-0.19)	0.12	(2.29)	-0.05	(-0.05)	-1.08	(-0.61)	0.13	(2.52)
Commodity	-1.04	(-1.03)	1.08	(8.08)	0.01	(0.01)	-0.07	(-0.04)	1.10	(8.19)
REIT	-2.49	(-2.05)	0.26	(2.11)	-0.75	(-0.57)	-6.76	(-3.34)	0.30	(2.45)

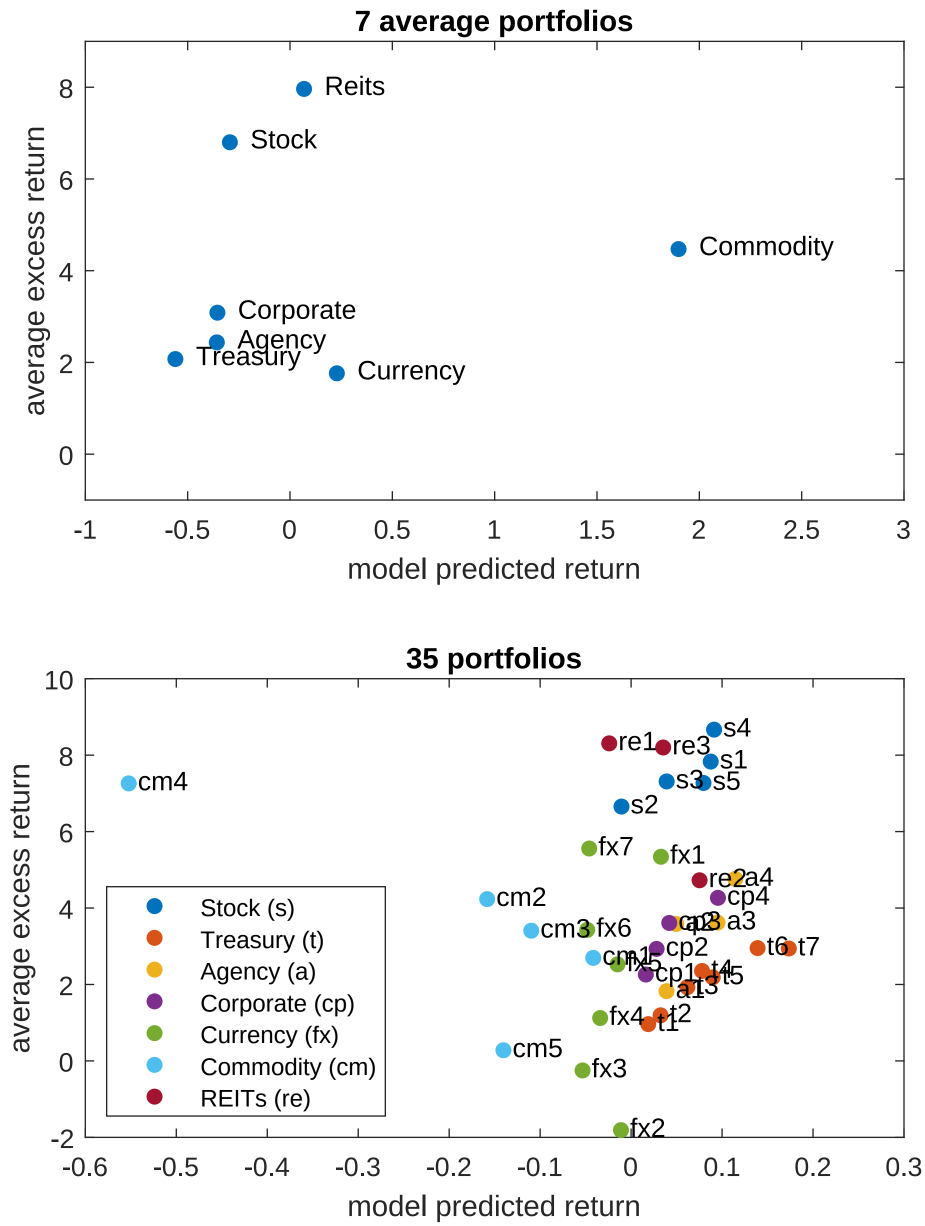
Notes: This table reports the inflation hedging properties of 7 average portfolios with respect inflation level, expected inflation, and inflation shock (headline, core, and energy). The t-statistics are in the parenthese.

Figure 1: Inflation



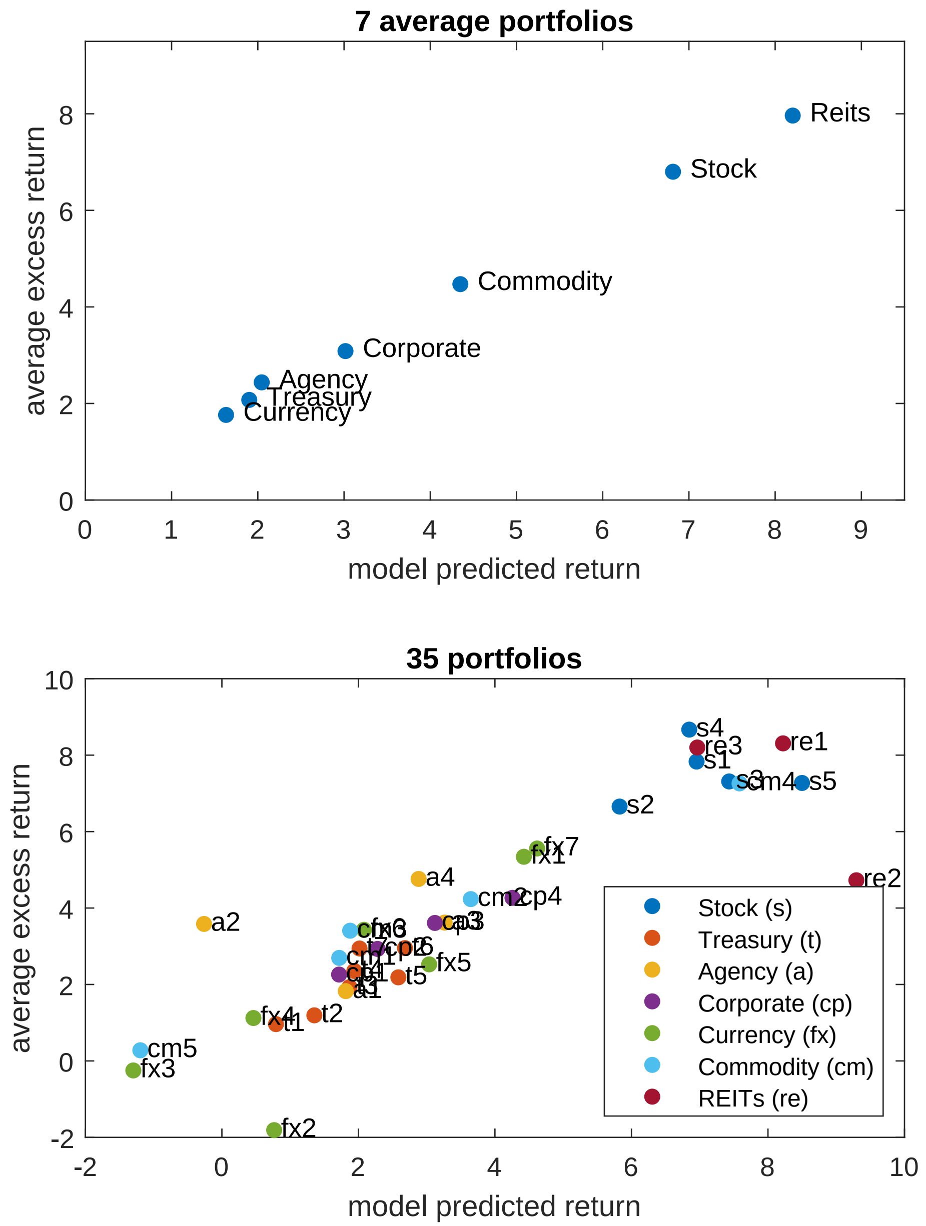
Notes: This figure plots the time-series of headline, core, food, and energy inflation shocks extracted from VAR described in the main text. Data are quarterly from 1963Q3 to 2019Q4.

Figure 2: Average Returns vs. Model Predicted Returns: Headline Inflation



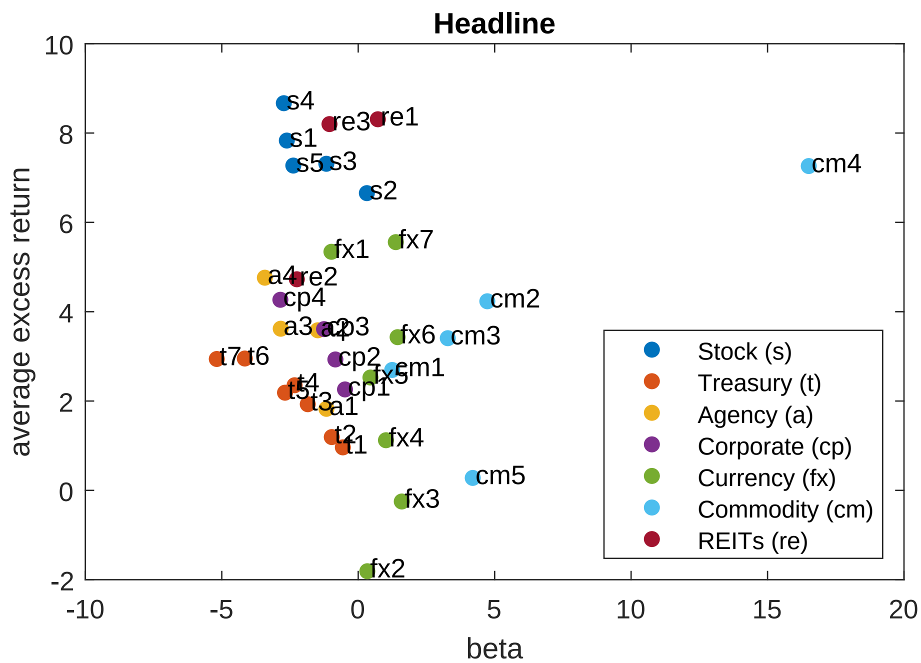
Notes: This figure plots the average excess return of the 7 average portfolios amd 35 portfolios against their model predicted returns with headline inflation only as the risk factor. The horizontal axis are model predicted returns, and the vertical axis shows the average excess returns. Each dot represents a portfolio. Abbreviation correspondence: s1 cons, s2 manu, s3 tech, s4 health, s5 others; t1-t7, a1-a4, cp1-cp4, from short to long maturity; fx1 dollar carry, fx2-fx7, from low interest rate to high interest rate; cm1 livestock, cm2 ind metal, cm3 pre metal, cm4 energy, cm5 agriculture; re1 equity, re2 mortgage, re3 hybrid.

Figure 3: Average Returns vs. Model Predicted Returns: Core and Energy Inflation



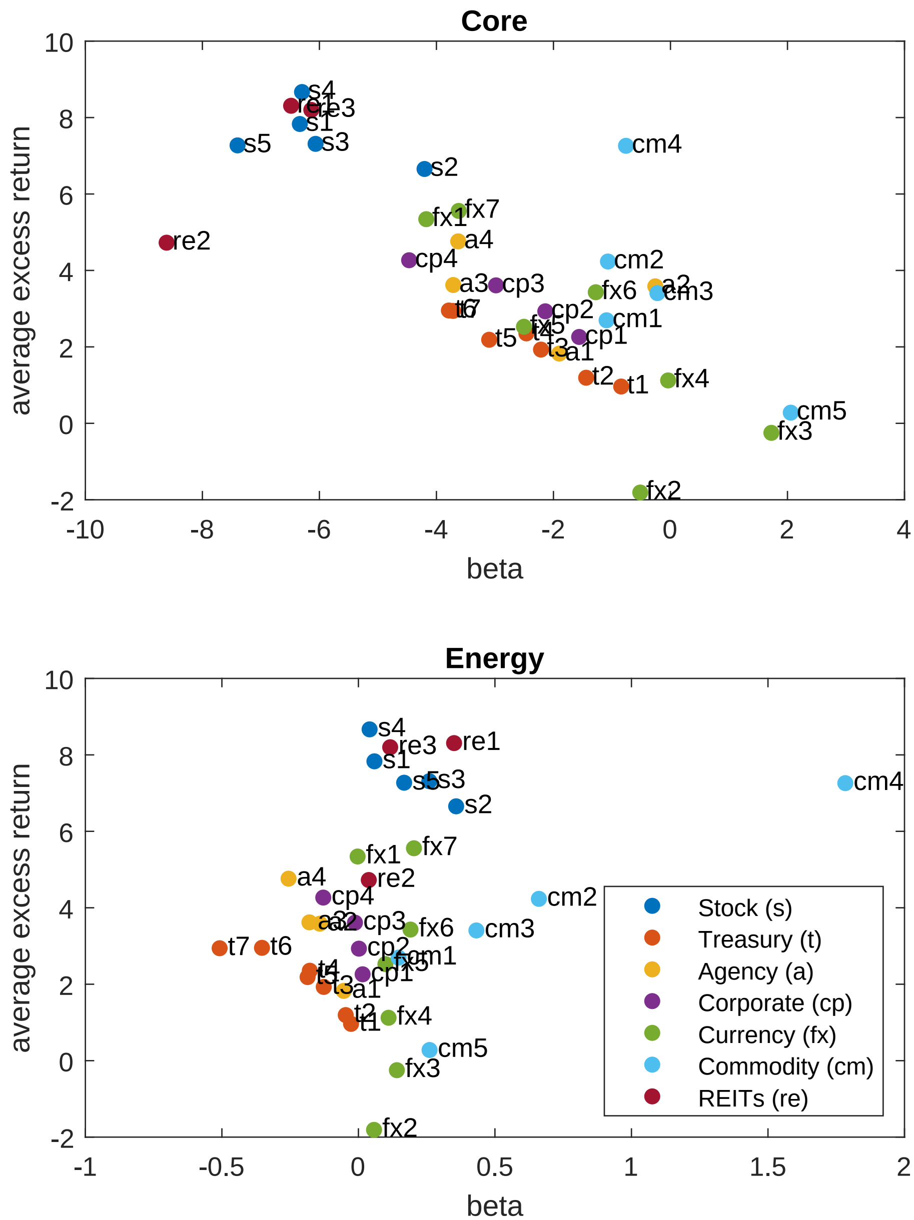
Notes: This figure plots the average excess return of the 7 average portfolios amd 35 portfolios against their model predicted returns with core and energy inflation risk factors. The horizontal axis are model predicted returns, and the vertical axis shows the average excess returns. Each dot represents a portfolio. Abbreviation correspondence: s1 cons, s2 manu, s3 tech, s4 health, s5 others; t1-t7, a1-a4, cp1-cp4, from short to long maturity; fx1 dollar carry, fx2-fx7, from low interest rate to high interest rate; cm1 livestock, cm2 ind metal, cm3 pre metal, cm4 energy, cm5 agriculture; re1 equity, re2 mortgage, re3 hybrid.

Figure 4: Average Returns and Headline Inflation Beta



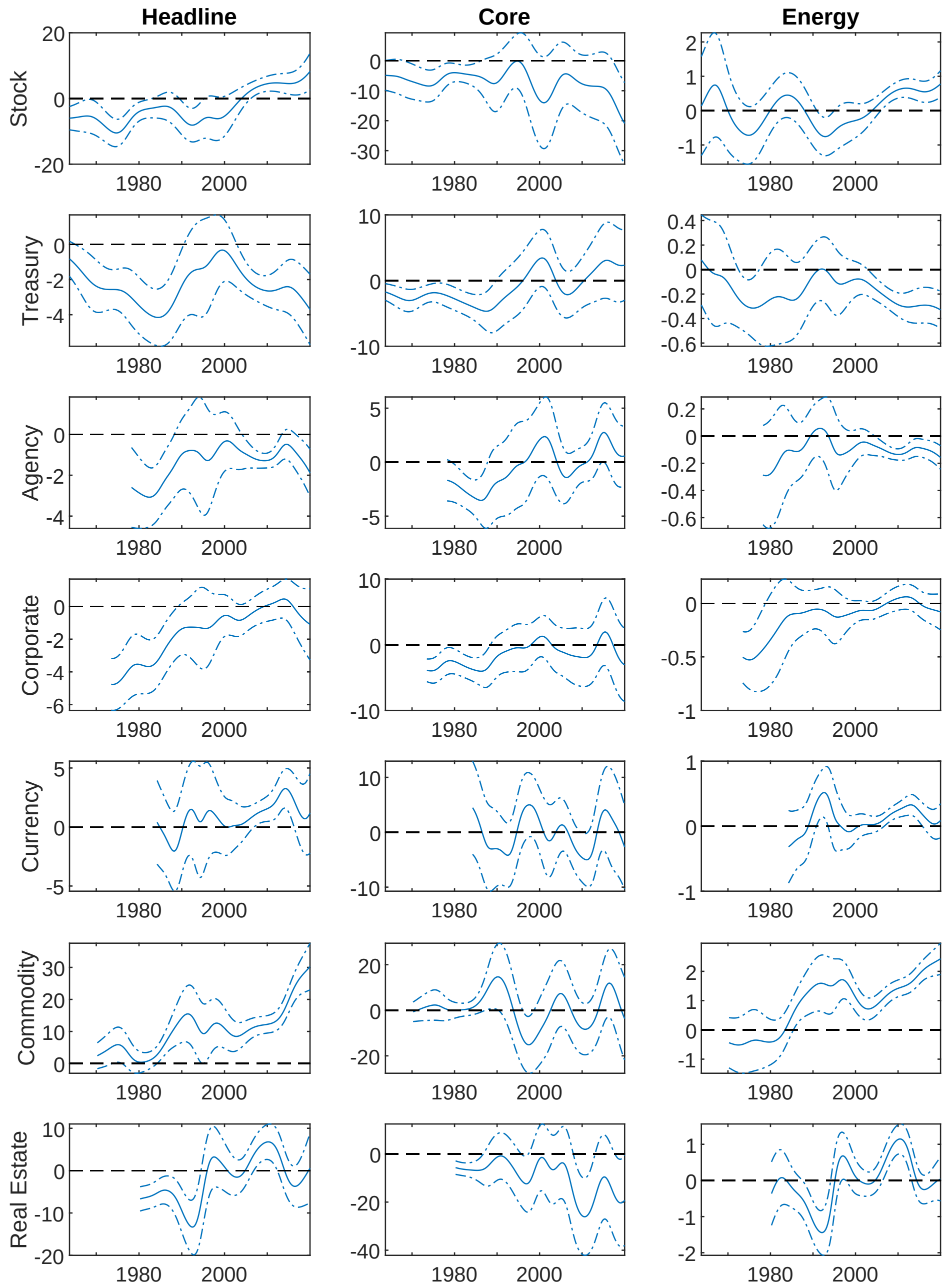
Notes: This figure plots the average excess return of the 35 test portfolios against their headline inflation betas. The horizontal axis shows headline inflation betas and the vertical axis shows the average excess returns. Each dot represents a test portfolio, and different colors refer to assets from different asset classes. Abbreviation correspondence: s1 cons, s2 manu, s3 tech, s4 health, s5 others; t1-t7, a1-a4, cp1-cp4, from short to long maturity; fx1 dollar carry, fx2-fx7, from low interest rate to high interest rate; cm1 livestock, cm2 ind metal, cm3 pre metal, cm4 energy, cm5 agriculture; re1 equity, re2 mortgage, re3 hybrid.

Figure 5: Inflation Beta and Average Returns



Notes:This figure plots the average excess return of the 35 test portfolios against their core and energy inflation betas (bivariate). The horizontal axis shows core and energy inflation betas and the vertical axis shows the average excess returns. Each dot represents a test portfolio, and different colors refer to assets from different asset classes. Abbreviation correspondence: s1 cons, s2 manu, s3 tech, s4 health, s5 others; t1-t7, a1-a4, cp1-cp4, from short to long maturity; fx1 dollar carry, fx2-fx7, from low interest rate to high interest rate; cm1 livestock, cm2 ind metal, cm3 pre metal, cm4 energy, cm5 agriculture; re1 equity, re2 mortgage, re3 hybrid.

Figure 6: Inflation Beta and Average Returns



Notes: This figure plots the time-varying estimates of betas. The core and energy betas are in a bi-variate regression. The estimates are from a Gaussian kernel estimator with bandwidth of 0.05. The 90% confidence intervals are plotted.

A Proofs

A.1 Proof of Proposition 1

The model has three exogenous state variables and no endogenous state variable. After log-linearization, all variables can be written as linear functions of the three state variables. So we can postulate:

$$c_{c,t} = c_{c,\mu}\mu_t + c_{c,e}c_{e,t} + c_{c,\delta}\delta_t, \pi_t = \pi_\mu\mu_t + \pi_e c_{e,t} + \pi_\delta\delta_t.$$

We plug in the postulated solution of core output and inflation into the equations (21) and (23):

$$\pi_\mu\mu_t+\pi_e c_{e,t}+\pi_\delta\delta_t = \beta(\rho_\mu\pi_\mu\mu_t+\rho_e\pi_e c_{e,t}+\rho_\delta\pi_\delta\delta_t)+\lambda[\kappa_c(c_{c,\mu}\mu_t+c_{c,e}c_{e,t}+c_{c,\delta}\delta_t)+\kappa_e c_{e,t}+\kappa_\delta\delta_t]+\lambda\mu_t$$

$$-\eta_c[(\rho_\mu-1)c_{c,\mu}\mu_t + (\rho_e-1)c_{c,e}c_{e,t} + (\rho_\delta-1)c_{c,\delta}\delta_t] - \kappa_e[(\rho_e-1)c_{e,t} + (\rho_\delta-1)\delta_t]$$

$$-(\rho_\mu-\phi_\pi)\pi_\mu\mu_t - (\rho_e-\phi_\pi)\pi_e c_{e,t} - (\rho_\delta-\phi_\pi)\pi_\delta\delta_t = 0$$

These two equations need to hold for all values of μ_t, so that:

$$(1-\beta\rho_\mu)\pi_\mu - \lambda\kappa_c c_{c,\mu} - \lambda = 0,$$

$$\eta_c(1-\rho_\mu)c_{c,\mu} + (\phi_\pi-\rho_\mu)\pi_\mu = 0.$$

We can solve for $c_{c,\mu}$ and π_μ from the two equations:

$$c_{c,\mu} = \frac{-\lambda(\phi_\pi-\rho_\mu)}{\eta_c(1-\rho_\mu)(1-\beta\rho_\mu)+\lambda\kappa_c(\phi_\pi-\rho_\mu)},$$

$$\pi_\mu = -\frac{\eta_c(1-\rho_\mu)}{\phi_\pi-\rho_\mu}c_{c,\mu} = \frac{\lambda\eta_c(1-\rho_\mu)}{\eta_c(1-\rho_\mu)(1-\beta\rho_\mu)+(\phi_\pi-\rho_\mu)\lambda\kappa_c}.$$

Since $\phi_\pi > 1, \rho_\mu < 1$, and recall that $\eta_c > 0, \lambda > 0, \kappa_c > 0$,we can easily see $c_{c,\mu} < 0, \pi_\mu > 0$.

Similarly, we can derive the conditions under which the two equations hold for all values of energy supply shock, $c_{e,t}$.

$$(1-\beta\rho_e)\pi_e - \lambda\kappa_c c_{c,e} - \lambda\kappa_e = 0,$$

$$\eta_c(1-\rho_e)c_{c,e} + \kappa_e(1-\rho_e) + (\phi_\pi-\rho_e)\pi_e = 0.$$

We can solve for $c_{c,e}$ andπ_e from the two equations:

$$c_{c,e} = \frac{-\kappa_e \left[1 - \rho_e + \frac{\lambda(\phi_\pi - \rho_e)}{1-\beta\rho_e}\right]}{\eta_c(1-\rho_e) + \frac{\lambda\kappa_c(\phi_\pi-\rho_e)}{1-\beta\rho_e}}$$

$$\pi_e = \frac{\lambda}{1-\beta\rho_e}(\kappa_e + \kappa_c c_{c,e}) = \frac{\lambda}{1-\beta\rho_e}\frac{\kappa_e(\eta_c - \kappa_c)(1-\rho_e)}{\eta_c(1-\rho_e) + \frac{\lambda\kappa_c(\phi_\pi-\rho_e)}{1-\beta\rho_e}} = -\frac{\lambda\kappa_e(1-\rho_e)\frac{\varphi+\alpha}{1-\alpha}}{(1-\beta\rho_e)\left[\eta_c(1-\rho_e) + \frac{\lambda\kappa_c(\phi_\pi-\rho_e)}{1-\beta\rho_e}\right]}$$

Since $\kappa_e < 0$, it is straightforward to see that $c_{c,e} > 0, \pi_e > 0$.

For the energy demand shock δ_t, we can similarly solve for $c_{c,\delta}$ and π_δ. They have almost the same expression as the energy supply shock except potential different persistence. We skip the derivation here.

A.2 Proof of Proposition 2

The real stochastic discount factor in comsumption basket is equal to $M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$, where C_t is the consumption basket. After log-linearization, we have:

$$m_{t+1} = -\gamma(c_{t+1} - c_t) = -\gamma\left[\hat{\alpha}_c(c_{c,t+1} - c_{c,t}) + (1-\hat{\alpha}_c)(c_{e,t+1} - c_{e,t} + \delta_{t+1} - \delta_t)\right]$$

$$= -\gamma[\hat{\alpha}_c c_{c,\mu}(\rho_\mu - 1)\mu_t + (\hat{\alpha}_c c_{c,e} + 1 - \hat{\alpha}_c)(\rho_e - 1)c_{e,t} + (\hat{\alpha}_c c_{c,\delta} + 1 - \hat{\alpha}_c)(\rho_\delta - 1)\delta_t]$$

$$= (-\gamma\hat{\alpha}_c c_{c,\mu})\sigma_\mu\varepsilon_{\mu,t+1} + [-\gamma(\hat{\alpha}_c c_{c,e} + 1 - \hat{\alpha}_c)]\sigma_e\varepsilon_{e,t+1} + [-\gamma(\hat{\alpha}_c c_{c,\delta} + 1 - \hat{\alpha}_c)]\sigma_\delta\varepsilon_{\delta,t+1}$$

If we write the SDF as $m_{t+1} = m_\mu(1-\rho_\mu)\mu_t + m_e(1-\rho_e)c_{e,t} + m_\delta(1-\rho_\delta)\delta_t - \lambda_\mu\sigma_\mu\varepsilon_{\mu,t+1} - \lambda_e\sigma_e\varepsilon_{e,t+1} - \lambda_\delta\sigma_\delta\varepsilon_{\delta,+1}$, we can obtain:

$$m_\mu = -\gamma\hat{\alpha}_c c_{c,\mu}(\rho_\mu - 1) < 0, m_e = -\gamma(\hat{\alpha}_c c_{c,e} + 1 - \hat{\alpha}_c)(\rho_e - 1) > 0,$$

$$m_\delta = -\gamma(\hat{\alpha}_c c_{c,e} + 1 - \hat{\alpha}_c)(\rho_\delta - 1) > 0,$$

$$\lambda_\mu = \gamma\hat{\alpha}_c c_{c,\mu} < 0, \lambda_e = \gamma(\hat{\alpha}_c c_{c,e} + 1 - \hat{\alpha}_c) > 0, \lambda_\delta = \gamma(\hat{\alpha}_c c_{c,\delta} + 1 - \hat{\alpha}_c) > 0.$$

All signs directly follow Proposition 1.

A.3 Proof of Lemma 3.2

$$d_t = \frac{1}{1-(1-\alpha)\exp(-\bar{\mu})}[c_{c,t} - (1-\alpha)\exp(-\bar{\mu})(w_t - p_t + n_t)] - p_{h,t}$$

$$= \frac{1}{1-(1-\alpha)\exp(-\bar{\mu})}[c_{c,t}-(1-\alpha)\exp(-\bar{\mu})(\gamma[(\hat{\alpha}_c c_{c,t}+(1-\hat{\alpha}_c)(c_{e,t}+\delta_t)]+\frac{\varphi+1}{1-\alpha}c_{c,t})]-\frac{1-\hat{\alpha}_c}{\phi}(c_{c,t}-c_{e,t}-\delta_t)$$

Therefore:

$$d_\mu = \left[\frac{1-(1-\alpha)\exp(-\bar{\mu})(\gamma\hat{\alpha}_c+\frac{\varphi+1}{1-\alpha})}{1-(1-\alpha)\exp(-\bar{\mu})} - \frac{1-\hat{\alpha}_c}{\phi}\right] c_{c,\mu}$$

$$d_e = \left[\frac{1-(1-\alpha)\exp(-\bar{\mu})(\gamma\hat{\alpha}_c+\frac{\varphi+1}{1-\alpha})}{1-(1-\alpha)\exp(-\bar{\mu})} - \frac{1-\hat{\alpha}_c}{\phi}\right] c_{c,e} - \frac{(1-\alpha)\exp(-\bar{\mu})\gamma(1-\hat{\alpha}_c)}{1-(1-\alpha)\exp(-\bar{\mu})} + \frac{1-\hat{\alpha}_c}{\phi}$$

$$d_\delta = \left[\frac{1-(1-\alpha)\exp(-\bar{\mu})(\gamma\hat{\alpha}_c+\frac{\varphi+1}{1-\alpha})}{1-(1-\alpha)\exp(-\bar{\mu})} - \frac{1-\hat{\alpha}_c}{\phi}\right] c_{c,\delta} - \frac{(1-\alpha)\exp(-\bar{\mu})\gamma(1-\hat{\alpha}_c)}{1-(1-\alpha)\exp(-\bar{\mu})} + \frac{1-\hat{\alpha}_c}{\phi}$$

Under Assumption 2, and $\hat{\alpha}_c$ being close to 1, we have $1-(1-\alpha)\exp(-\bar{\mu})(\gamma+\frac{\varphi+1}{1-\alpha}) > 0$. Therefore:

$$d_\mu < 0, d_e > 0, d_\delta > 0.$$

A.4 Proof of Proposition 3

Note that according to Campbell-Shiller decompsition, the return to core stock can be written as:

$$r_{s,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta d_{t+1},$$

where z_t is the log price-dividend ratio of the core stock. Postulate that $z_t = z_\mu \mu_t + z_e c_{e,t} + z_\delta \delta_t$, then the return to core stock is equal to:

$$r_{s,t+1} = \kappa_0 + \kappa_1(z_\mu \mu_{t+1} + z_e c_{e,t+1} + z_\delta \delta_{t+1}) - (z_\mu \mu_t + z_e c_{e,t} + z_\delta \delta_t) + \Delta d_{t+1}$$

We solve for the coefficients z_μ, z_e, z_δ from the Euler equation $E_t(m_{t+1}+r_{s,t+1})+\frac{1}{2}var(m_{t+1}+r_{s,t+1}) = 0$.

$$E_t r_{s,t+1} = \kappa_0+[(\kappa_1\rho_\mu-1)z_\mu+(\rho_\mu-1)d_\mu]\mu_t+[(\kappa_1\rho_e-1)z_e+(\rho_e-1)d_e]c_{e,t}+[(\kappa_1\rho_\delta-1)z_\delta+(\rho_\delta-1)d_\delta]\delta_t$$

And:

$$E_t m_{t+1} = m_\mu(1-\rho_\mu)\mu_t + m_e(1-\rho_e)c_{e,t} + m_\delta(1-\rho_\delta)\delta_t$$

Note that $var_t(m_{t+1} + r_{s,t+1})$ is a constant and the Euler equation has to hold for all values of state variables μ_t, we have the following:

$$(\kappa_1\rho_\mu - 1)z_\mu + (\rho_\mu - 1)d_\mu + (1 - \rho_\mu)m_\mu = 0$$

We can solve for z_μ as:

$$z_\mu = \frac{1 - \rho_\mu}{1 - \kappa_1\rho_\mu}(m_\mu - d_\mu)$$

Similarly, the following equation should hold so that the Euler equation holds for all values of $c_{e,t}$:

$$(\kappa_1\rho_e - 1)z_e + (\rho_e - 1)d_e + (1 - \rho_e)m_e = 0$$

Solve for z_e, z_δ as:

$$z_e = \frac{1 - \rho_e}{1 - \kappa_1\rho_e}(m_e - d_e), z_\delta = \frac{1 - \rho_\delta}{1 - \kappa_1\rho_\delta}(m_\delta - d_\delta)$$

Then we plug in the log price-dividend ratio into the Campbell-Shiller decomposition and solve for the core stock return:

$$r_{s,\mu} = \kappa_1 z_\mu + d_\mu = \frac{\kappa_1(1 - \rho_\mu)}{1 - \kappa_1\rho_\mu}(m_\mu - d_\mu) + d_\mu = \frac{\kappa_1(1 - \rho_\mu)}{1 - \kappa_1\rho_\mu}m_\mu + \frac{1 - \kappa_1}{1 - \kappa_1\rho_\mu}d_\mu < 0,$$

$$r_{s,e} = \frac{\kappa_1(1 - \rho_e)}{1 - \kappa_1\rho_e}m_e + \frac{1 - \kappa_1}{1 - \kappa_1\rho_e}d_e > 0, r_{s,\delta} = \frac{\kappa_1(1 - \rho_\delta)}{1 - \kappa_1\rho_\delta}m_\delta + \frac{1 - \kappa_1}{1 - \kappa_1\rho_\delta}d_\delta > 0.$$

A.5 Proof of Proposition 4

We use the nominal SDF of the households to price the nominal bonds. Consider the two-period bond issued at time t at price $P_t^{(2)}$. At time $t + 1$, the bond price becomes $P_{t+1}^{(1)}$. Therefore, the following Euler equations hold for the two-pseriod bond and one-period bond:

$$E_t M_{t+1}^{\$} P_{t+1}^{(1)} = P_t^{(2)}, E_t M_{t+1}^{\$} = P_t^{(1)}.$$

Take logs on both sides:

$$p_t^{(1)} = p_0^{(1)} + E_t m_{t+1}^{\$}$$

$$p_t^{(2)} = p_0^{(2)} + E_t m_{t+1}^{\$} + E_t p_{t+1}^{(1)}$$

where $p_0^{(1)}$ and $p_0^{(2)}$ are constants that originate from the second-order moments.

$$E_t m^{\$}_{t+1} = m_\mu(1-\rho_\mu)\mu_t + m_e(1-\rho_e)c_{e,t} + m_\delta(1-\rho_\delta)\delta_t$$

$$-\frac{1-\hat{\alpha}_c}{\phi}E_t(c_{c,t+1} - c_{c,t} - c_{e,t+1} + c_{e,t} - \delta_{t+1} + \delta_t) - E_t\pi_{t+1}$$

We rewrite $E_t m^{\$}_{t+1} = m^{\$}_\mu \mu_t + m^{\$}_\delta \delta_t + m^{\$}_e c_{e,t}$, where

$$m^{\$}_\mu = m_\mu(1-\rho_\mu) + \frac{1-\hat{\alpha}_c}{\phi}(1-\rho_\mu)c_{c,\mu} - \rho_\mu \pi_\mu,$$

$$m^{\$}_e = m_e(1-\rho_e) + \frac{1-\hat{\alpha}_c}{\phi}[(1-\rho_e)(c_{c,e}-1)] - \rho_e\pi_e, m^{\$}_\delta = m_\delta(1-\rho_\delta) + \frac{1-\hat{\alpha}_c}{\phi}[(1-\rho_\delta)(c_{c,\delta}-1)] - \rho_\delta\pi_\delta.$$

As $m_\mu < 0, c_{c,\mu} < 0, \pi_\mu > 0$, it is straightforward that $m^{\$}_\mu < 0$.

Recall that $m_e = \gamma(\hat{\alpha}_c c_{c,e} + 1 - \hat{\alpha}_c)$, thus
$m^{\$}_e = (1-\rho_e)\left[m_e + \frac{1-\hat{\alpha}_c}{\phi}(c_{c,e}-1)\right] = (1-\rho_e)(\eta_c c_{c,e} + \kappa_e) - \rho_e\pi_e < 0$

The last inequality comes from that fact that $\eta_c c_{c,e} + \kappa_e < 0$, which is from the proof of Proposition 1. Similarly, $m^{\$}_\delta < 0$.

The return to a long-term bond is equal to $r_{b,t+1} = p^{(1)}_{t+1} - p^{(2)}_t = const + E_{t+1}m^{\$}_{t+2} - (E_t m^{\$}_{t+1} + E_t p^{(1)}_{t+1})$. We decompose $r_{b,t+1}$ into predictable component $r_{b0,t}$ and unpredictable component $r_{b,\mu}\sigma_\mu\varepsilon_{t+1} + r_{b,e}\sigma_e\varepsilon_{e,t+1} + r_{b,\delta}\sigma_\delta\varepsilon_{\delta,t+1}$, we have:

$$r_{b,\mu} = m^{\$}_\mu < 0, r_{b,\delta} = m^{\$}_\delta < 0, r_{b,e} = m^{\$}_e < 0.$$

A.6 Proof of Prosition 5

The exposure of nominal foreign currency return to the three shocks are:

$$r_{fx,\mu} = \lambda_\mu + \frac{1-\hat{\alpha}_c}{\phi}c_{c,\mu} + \pi_\mu = \gamma\hat{\alpha}_c c_{c,\mu} + \frac{1-\hat{\alpha}_c}{\phi}c_{c,\mu} - \frac{\left(\gamma\hat{\alpha}_c + \frac{1-\hat{\alpha}_c}{\phi}\right)(1-\rho_\mu)}{\phi_\pi - \rho_\mu}c_{c,\mu} = \frac{\phi_\pi - 1}{\phi_\pi - \rho_\mu}\left(\gamma\hat{\alpha}_c + \frac{1-\hat{\alpha}_c}{\phi}\right) c_c$$

$$r_{fx,e} = \lambda_e + \frac{1-\hat{\alpha}_c}{\phi}(c_{c,e}-1) + \pi_e = \left(\eta_c + \frac{\lambda\kappa_c}{1-\beta\rho_e}\right)c_{c,e} + \left(\frac{\lambda}{1-\beta\rho_e} + 1\right)\kappa_e$$

Though we cannot determine conclusively on the signs of $r_{fx,e}$, we can see the three forces. Real SDF's loading on energy supply shock is $\lambda_e > 0$. The relative headline price loading is $\frac{1-\hat{\alpha}_c}{\phi}(c_{c,e}-1) < 0$, and the core good inflation loading is $\pi_e > 0$. The signs of loadings on energy demand shock are similar.

A.7 Proof of Proposition 6

Commodity futures satisfy the following Euler equation:

$$E_t M_{t+1} \frac{P_{e,t+1}}{P_{h,t+1}} = E_t M_{t+1} F_t$$

Taking log on both sides:

$$f_t = f_0 + E_t(p_{e,t+1} - p_{h,t+1}) = f_0 + E_t\left[p_{e,t+1} - (1-\hat{\alpha}_c)(p_{e,t+1} - \delta_{t+1})\right]$$

$$= \frac{\hat{\alpha}_c}{\phi} c_{c,\mu} \rho_\mu \mu_t + \frac{\hat{\alpha}_c}{\phi}(c_{c,e} - 1)\rho_e c_{e,t} + \left[\hat{\alpha}_c \left(\frac{\phi - 1}{\phi} + \frac{1}{\phi} c_{c,\delta}\right) + (1-\hat{\alpha}_c)\right] \rho_\delta \delta_t$$

Straightforwardly, $f_\mu < 0, f_e < 0, f_\delta > 0$.

B Parameter Values

In this appendix, we show the parameter values we use in Section 3.5.

Table B1: Parameter Values

Variable	Symbol	Value
Substitution between core and energy	ϕ	2
IES	$\frac{1}{\gamma}$	3
Core expenditure share	$\hat{\alpha}_c$	0.9
Prob of price adjustment	$1-\theta$	$1/4$
Labor share	$1-\alpha$	$2/3$
Time discount rate	β	0.99
Average markup	$\exp(\bar{\mu})$	5
Taylor rule response	ϕ_π	1.5
Frisch elasticity of labor	φ	2
Energy supply persistence	ρ_e	0
Energy demand persistence	ρ_δ	0

C Additional Empirical Results

C.1 VAR Estimates

Table C1 reports the VAR coefficient matrix and their statistical significance.

Table C1: Shocks to Expected Inflation

	A. Risk Exposure		
	core	food	energy
core(-1)	0.46	0.16	1.74
	(7.41)	(1.11)	(2.15)
food(-1)	0.08	0.27	0.28
	(2.98)	(4.04)	(0.77)
inflation(-1)	0.01	0.02	-0.02
	(1.22)	(1.21)	(-0.29)
rf(-1)	0.15	0.01	0.00
	(3.01)	(0.05)	(-0.00)
pd(-1)	-1.23	-1.54	6.11
	(-3.19)	(-1.68)	(1.21)
output gap(-1)	0.06	0.32	0.31
	(1.31)	(2.90)	(0.50)
R^2	0.70	0.26	0.04

Notes: This table reports estimates of VAR(1). The t-statistics are in the parenthese. The VAR includes the core, food, and energy inflation, the risk-free rate, price-dividend ratio, and the output gap.

C.2 Alternative Construction of Inflation Shocks

In this subsection, we report the robustness of the inflation shocks to the way we extract them. We calculate the shocks to headline, core, and energy inflation from an AR(1), ARMA(1,1), and a VAR model (shown in the main text). The shock series constructed from the three models are highly correlated, with the correlation listed in Table C2. Therefore, our extracted shock series is robust to different models.

Table C2: Correlation of Shocks

	AR(1) and ARMA(1,1)	ARMA(1,1) and VAR	AR(1) and VAR
Headline	0.92	0.92	0.90
Core	0.93	0.86	0.89
Energy	1.00	0.97	0.98

Notes: This table shows the correlation of headline, core, and energy inflation shocks constructed from AR(1), ARMA(1,1), and VAR models.

C.3 GMM Estimation

In this section, we report the standard errors of prices of inflation risks with GMM estimation. The moment conditions are written as:

$$E_T \begin{bmatrix} (1 - b'\varepsilon_{\pi,t})r_t \\ \lambda - b'\varepsilon_{\pi,t}\varepsilon_{\pi,t}^{'} \end{bmatrix} = 0$$

where $m_t = 1 - b'\varepsilon_{\pi,t}$ is the stochastic discount factor, $\varepsilon_{\pi,t}$ is the corresponding inflation risk factor, λ is the price of risk, and E_T is the operator of time-series average.

Table C3 reports the estimation results. These estimates are identical in sign and significance with the estimates obtained using the two-step Fama-MacBeth approach and very similar in magnitude.

Table C3: Price of Inflation Risks

	A. 7 Average Portfolios			B. 35 Portfolios	
headline λ	0.20		-0.07		
t-stat	(0.68)		(-0.29)		
core λ		-1.08		-1.06	-0.65
t-stat		(-3.02)		(-3.88)	(-2.93)
energy λ		3.88		3.80	0.01
t-stat		(1.36)		(1.78)	(0.04)

Notes: This table reports the price of risk estimated using the GMM. Panel A uses the 7 average portfolios from each asset class as test portfolios. Panel B uses the 35 portfolios as. In each panel, the first column reports the price of headline inflation and the second column reports the price of core and energy inflation. Standard errors are Newey-West adjusted.

C.4 Betas at Lower Frequency

In this section, we report empirical results with low-frequency betas. This approach follows Bansal et al. (2005) and Lettau et al. (2019). In the first-step of regression, we regress the cumulative asset return from quarter $t-\tau$ to quarter t on the unexpected inflation from quarter $t-\tau$ to quarter t.

$$r_{t-\tau,t} = a + \beta\varepsilon_{\pi,t-\tau,t} + u_t$$

The unexpected inflation over the τ quarters is computed from the VAR system $Y_t = c + \Phi Y_{t-1} + u_t$.

$$\begin{aligned}\varepsilon_{\pi,t-\tau,t} &= \sum_{j=0}^{\tau-1}(Y_{t-j} - E_{t-\tau}Y_{t-j}) = \sum_{j=0}^{\tau-1} Y_{t-j} - hc[(I-A)^{-1}(I-A^{\tau})] - \sum_{j=0}^{\tau-1} Y_{t-\tau}A^{\tau-j} \\ &= \sum_{j=0}^{\tau-1} Y_{t-j} - \tau c[(I-A)^{-1}(I-A^{\tau})] - Y_{t-\tau}[(I-A)^{-1}(I-A^{\tau+1}) - I]\end{aligned}$$

$\varepsilon_{\pi,t-\tau,t}$ are the corresponding rows of the vector. The covariation over longer-horizon captures the low-frequency relation between inflation and asset prices. $r_{t-\tau,t}$ is the cumulative asset excess return from quarter $t-\tau$ to t. We select $\tau = 8$ and examine the covariation at the eight-quarter frequency. Table C4 reports the β's prices of risks using the 7 average portfolios .

The asset loadings and price of risks are largely identical with the ones presented in the main text. The only difference is that the price of energy inflation is positive and statistically significant. One potential reason is that energy inflation is quite noisy at higher frequency, which contaminates the beta estimates and hinders the discovery of its price of risk. When we look at the lower-frequency covariation, energy inflation carries a positive risk premium, i.e., a higher energy inflation indicates good news for investors.

Table C4: Eight-Quarter Inflation Exposure and Price of Risk

	core	t-stat	energy	t-stat
	A. Asset Return Exposure			
Stock	-4.17	(-3.19)	0.40	(1.41)
Treasury	-1.45	(-2.16)	-0.20	(-2.77)
Agency	-2.10	(-6.94)	-0.11	(-2.57)
Corporate	-2.62	(-4.12)	-0.04	(-0.37)
Currency	-3.01	(-1.39)	0.24	(1.98)
Commodity	-4.64	(-3.09)	2.00	(6.07)
REIT	-1.76	(-0.59)	0.76	(1.77)
	B. Price of Risk			
	core	t-stat	energy	t-stat
7 portfolios	-1.16	(-2.98)	1.56	(0.77)
35 portfolios	-1.07	(-3.03)	3.13	(1.35)

Notes: This table reports the two-step asset pricing results for core and energy inflation. Panel A reports the asset return exposures for the 7 average portfolios. The betas are computed by regressing 8-quarter cumulative excess returns on 8 quarter inflation shocks. Panel B reports the price of risk estimates for both 7 portfolios and 35 portfolios. The t-statistics are in the parenthese.

C.5 Shocks to Expected Inflation

This section reports the asset pricing test results for 7 average portfolios with respect to the shock to expected core inflation. The shock to expected core inflation is constructed as $A\varepsilon_{\pi,t}$, where A is the coefficient matrix in the VAR. The shock to expected core inflation is highly correlated with the shock to core inflation itself, so the asset pricing rests are similar, too. The correlation between shocks to core inflation and shocks to expected core inflation is 0.90.

Table C5: Shocks to Expected Inflation

	A. Risk Exposure			
	shock to core expectation	t-stat	energy shock	t-stat
Stock	-14.74	(-6.10)	0.36	(3.13)
Treasury	-5.03	(-5.21)	-0.16	(-3.50)
Agency	-4.98	(-5.78)	-0.05	(-1.49)
Corporate	-6.53	(-6.71)	0.01	(0.35)
Currency	-3.19	(-1.20)	0.16	(2.72)
Commodity	2.66	(0.86)	1.06	(7.61)
REIT	-14.06	(-4.16)	0.44	(3.44)
	C. Price of Risk			
	shock to core expectation	t-stat	energy shock	t-stat
7 portfolios	-0.40	(-2.88)	4.87	(1.81)
35 portfolios	-0.41	(-3.50)	4.21	(1.63)

Notes: This table reports the two-step asset pricing results for shock to core inflation expectation and energy shock. Panel A reports the asset return exposures for the 7 average portfolios, and Panel B reports the price of risk estimates for both 7 portfolios and 35 portfolios. The t-statistics are in the parenthese.

C.6 Maximum Correlation Portfolios

Panel A through C of Table C6, we report the same statistics of the maximum correlation portfolios. To construct the maximum correlation portfolios, we regress the inflation factors onto the available assets and use the regression coefficients as portfolio weights. The maximum correlation portfolios have the maximum correlation with the test assets that are used for constructing the mimicking portfolios.The average return of mimicking portfolios from different asset classes are largely consistent except for currencies and commodities, as these two assets classes are not exposed to the core inflation. The average returns for headline and energy inflation mimicking portfolios do not have a clear pattern.

Table C6: Mimicking Portfolios: Maximum Correlation Portfolios

	Stock	Treasury	Agen	Corp	Curr	Comm	REITs	Aver	All
				A. Core					
mean	-0.18	-0.16	-0.13	-0.14	0.00	0.00	-0.03	-0.06	-0.17
t-stat	(-3.99)	(-2.46)	(-3.11)	(-2.71)	(-0.12)	(-0.01)	(-1.71)	(-4.10)	(-4.87)
Sharpe ratio	-0.54	-0.35	-0.40	-0.41	-0.02	0.00	-0.29	-0.62	-0.70
$corr(r_{fmp}, \pi)$	0.47	0.67	0.45	0.49	0.40	0.29	0.31	0.24	0.67
				B. Energy					
mean	1.03	-1.55	-1.62	2.25	0.06	1.19	0.14	0.39	-2.34
t-stat	(1.38)	(-2.17)	(-2.85)	(3.01)	(0.05)	(1.12)	(0.16)	(0.31)	(-1.55)
Sharpe ratio	0.19	-0.26	-0.42	0.39	0.01	0.18	0.03	0.05	-0.27
$corr(r_{fmp}, \pi)$	0.57	0.57	0.36	0.54	0.58	0.58	0.42	0.66	0.76
				C. Headline					
mean	-0.18	-0.33	-0.27	0.07	-0.01	0.11	0.01	-0.04	-0.38
t-stat	(-1.98)	(-3.08)	(-3.32)	(0.73)	(-0.04)	(1.00)	(0.15)	(-0.29)	(-2.52)
Sharpe ratio	-0.27	-0.44	-0.48	0.09	-0.01	0.17	0.03	-0.05	-0.43
$corr(r_{fmp}, \pi)$	0.55	0.63	0.47	0.59	0.57	0.57	0.41	0.64	0.78

Notes: The table reports the characteristics the maximum correlation portfolios. The table reports the mean, the t-statistics, the Sharpe ratio, and the correlation between the portfolio and the corresponding inflation factor. The columns indicate the test assets used to construct the mimicking portfolios.

D The Extended Model

In this section, we present an extended version of the model in the main text. All asset pricing implications of the model carries through to the extended one. The goal of the extension is to break down the consumption CAPM. This way, the model is consistent with the empirical fact that controlling for consumption growth does not drive out the risk premium of core inflation.

D.1 Model Setting

The extended model is similar with the one in the main text in the preference on consumption, production technology, price stickiness and monopolistic competitive goods market structure. The only difference is that there are two types of agents, workers (fraction θ_w) and shareholders (fraction $1 - \theta_w$). Workers supply labor and do not participate in the financial market. Shareholders own the equity claims of core firms. Energy goods are prorataly endowed. Variables with superscript w are associated with workers, and those with superscipt e are associated with shareholders. We briefly outline the equilibrium conditions here.

D.1.1 Workers

The consumption-labor marginal optimality condition for workers is:

$$\frac{(C_t^w)^{-\gamma}}{P_t P_{ht}} = \frac{N_t^\varphi}{W_t}$$

D.1.2 Shareholders

The Euler equations for shareholders are:

$$E_t \beta \left(\frac{C_{t+1}^s}{C_t^s} \right)^{-\gamma} \frac{P_{ht} P_t}{P_{h,t+1} P_{t+1}} (1 + i_t) = 1$$

$$E_t \beta \left(\frac{C_{t+1}^s}{C_t^s} \right)^{-\gamma} \frac{P_{ht} P_t}{P_{h,t+1} P_{t+1}} \frac{D_{t+1} + P_{s,t+1}}{P_{s,t}} = 1$$

D.1.3 Consumption Aggregation

For both workers and shareholders ($i = w, s$), the consumption basket is defined as:

$$C^i = \left[\alpha_c (C_c^i)^{\frac{\phi-1}{\phi}} + (1-\alpha_c)\left[\exp(\delta) C_e^i\right]^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}$$

The relative price of energy satisfies:

$$P_e = \frac{1-\alpha_c}{\alpha_c}\left(\frac{C_e^i}{C_c^i}\right)^{-\frac{1}{\phi}} \exp\left(\frac{\phi-1}{\phi}\delta\right)$$

The headline price satisfies:

$$P_h = \left\{\alpha_c^{\phi} + (1-\alpha_c)^{\phi}\left[\exp(-\delta) P_e\right]^{1-\phi}\right\}^{\frac{1}{1-\phi}}$$

D.1.4 Productive Core Firms

Each core firm produces one variety of core good and each firm is monopolistic in the specific variety production. All varieties are aggregated into core consumption in a CES manner, with elasticity of substitution ϕ. Production technology for variety j is:

$$Y_t(j) = A N_t(j)^{1-\alpha}$$

The marginal cost of production, similar with the model in the main text, is:

$$MC(Y) = \frac{W}{P}\frac{1}{(1-\alpha)Y}\left(\frac{Y}{A}\right)^{\frac{1}{1-\alpha}}$$

Since the aggregator labor supply in the economy is equal to $\theta_w N$, we normalize the aggregate TFP $A = \theta_w^{-(1-\alpha)}$. The New Keynesian Phillips Curve is written as

$$\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\frac{u'(C_{t+k}^s)}{u'(C_t^s)}\frac{P_{ht}}{P_{h,t+1}}\left[Y_{t+k|t} + (P_t^* - \Psi'(Y_{t+k|t}))\frac{\partial Y_{t+k|t}}{\partial P_t^*}\right]\right\} = 0$$

where:

$$\frac{\partial Y_{t+k|t}}{\partial P_t^*} = -\varepsilon_{t+k}\left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon_{t+k}}\frac{C_{C,t+k}}{P_t^*} = -\varepsilon_{t+k}\frac{Y_{t+k|t}}{P_t^*}$$

D.1.5 Market Clearing Conditions

There are two types of agents, so we need to include the aggregate budget constraint of one type of agent in the system of equations that consist of the equilibrium.

$$\theta_w(C_{ct}^w + P_{et}C_{et}^w) = \frac{W_t}{P_t}N_t\theta_w + P_{et}\theta_w Q_t$$

The two market clearing conditions are:

$$\theta_w C_c^w + (1-\theta_w) C_c^s = Y$$

$$\theta_w C_e^w + (1-\theta_w) C_e^s = C_e$$

D.1.6 Monetary Policy

The monetary policy follows a Taylor rule:

$$i_t = \bar{i} + \phi_\pi \pi_t$$

D.2 Log-linearization

When we log-linearize the system of equations, we make a parametric assumption to keep the algebra simplified: $\theta_w = (1-\alpha)\exp(-\bar{\mu})$. At the steady state, labor income share is equal to $(1-\alpha)\exp(-\bar{\mu})$. With our parametric assumption, the fraction of workers is equal to the steady state labor income share, the per capital consumption of core and energy goods are identical for workers and entrepreneurs at the steady state.

In the extended model, we make an additional parametric assumption that $\hat{\alpha}_c \to 1$. In the data energy inflation only accounts for about 10 percent of the headline inflation. This assumption can greatly simplify algebra in deriving the solutions.

The equilibrium of the economy satisfies the following log-linearized three-equation system with three unknowns: $c^s_{ct}, c^s_{et}, \pi_t$. The three equations are the Phillips curve, the Euler equation of the shareholders, and the workers' budget constraint.

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \mu_t + \lambda \left[\frac{\varphi+\alpha}{1-\alpha} y_t + \gamma \left(\hat{\alpha} c^w_{ct} + (1-\hat{\alpha}_c)(c^w_{et} + \delta_t) \right) + \frac{1-\hat{\alpha}_c}{\phi} (c^s_{ct} - c^s_{et} - \delta_t) \right]$$

$$-\gamma E_t \left[\hat{\alpha}_c (c^s_{c,t+1} - c^s_{ct}) + (1-\hat{\alpha}_c)(c^s_{e,t+1} - c^s_{et} + \delta_{t+1} - \delta_t) \right] - E_t \pi_{t+1}$$

$$-\frac{1-\hat{\alpha}_c}{\phi} E_t \Delta (c^s_{c,t+1} - c^s_{e,t+1} - \delta_{t+1}) + \phi_\pi \pi_t = 0$$

$$\hat{\alpha}_c c^w_{ct} + (1-\hat{\alpha}_c) c^w_{et} = \hat{\alpha}_c \left[\gamma (\hat{\alpha}_c c^w_{ct} + (1-\hat{\alpha}_c)(c^w_{et} + \delta_t)) + \frac{\varphi+1}{1-\alpha} y_t + \frac{1-\hat{\alpha}_c}{\phi} (c^s_{ct} - c^s_{et} - \delta_t) \right] + (1-\hat{\alpha}_c) c_{et}$$

where: $c^w_{ct} = \frac{1}{\theta_w}(c_{et} - c^s_{et}) + c^s_{ct}, c^w_{et} = \frac{1}{\theta_w}[c_{et} - (1-\theta_w) c^s_{et}], y_t = c_{et} - c^s_{et} + c^s_{ct}$, which can be straightforwardly derived from the relative price of energy good and the market clearing conditions.

D.3 Solution

We can express all the variables as linear functions of the three exogenous variables, μ_t, c_{et}, δ_t. Let:

$$c^s_{ct} = c_\mu \mu_t + c_e c_{et} + c_\delta \delta_t, c^s_{et} = e_\mu \mu_t + e_e c_{et} + e_\delta \delta_t, \pi_t = \pi_\mu \mu_t + \pi_e c_{et} + \pi_\delta \delta_t.$$

We keep the same assumption that the steady state level of markup is sufficiently large, and $\frac{1}{\gamma} > \phi > 1$.

D.3.1 Markup Shock

We can solve for the undetermined coefficients as follows:

$$c_\mu = \frac{\lambda}{(1-\beta\rho_\mu)y_\mu - \lambda\left[\frac{\varphi+\alpha}{1-\alpha}(1-x_\mu) + \gamma(1-\frac{x_\mu}{\theta_w})\right]}$$

where:

$$x_\mu = \theta_w + \frac{\frac{\varphi+1}{1-\alpha}(\theta_w - \theta_w^2)}{-1+\gamma+\frac{\varphi+1}{1-\alpha}\theta_w} < 0, y_\mu = -\frac{\gamma(1-\rho_\mu)}{\phi_\pi - \rho_\mu} < 0$$

$$e_\mu = x_\mu c_\mu, \pi_\mu = y_\mu c_\mu$$

Therefore, $c_\mu < 0, e_\mu > 0, \pi_\mu > 0$.

The core output loading on the markup shock is equal to $y_\mu = c_\mu - e_\mu = c_\mu(1-x) < 0$.

D.3.2 Energy Demand Shock

$$\pi_\delta = \frac{\frac{\varphi+\alpha}{1-\alpha}\frac{1}{\gamma}(\frac{1}{\phi}-\gamma)}{(\frac{\varphi+\alpha}{1-\alpha}\frac{1}{\gamma}+1)\frac{\phi_\pi-\rho_\delta}{1-\rho_\delta} + \frac{1-\beta\rho_\delta}{\lambda}(1-\frac{\gamma}{1-\theta_w} - \frac{\theta_w}{1-\theta_w}\frac{\varphi+\alpha}{1-\alpha})}(1-\hat{\alpha}_c) > 0$$

$$c_\delta = \frac{1}{\gamma}(\frac{1}{\phi}-\gamma)(1-\hat{\alpha}_c) - \frac{1}{\gamma}\frac{\phi_\pi-\rho_\delta}{1-\rho_\delta}\pi_\delta > 0$$

$$e_\delta = -\frac{1-\beta\rho_\delta}{\lambda}\frac{\theta_w}{1-\theta_w} < 0$$

Thus, core output loading is equal to $y_\delta = c_\delta - e_\delta > 0$

D.3.3 Energy Supply Shock

As in the main text, energy supply and demand shock plays exactly the same role, i.e.:

$$c_e > 0, \pi_e > 0, e_e - 1 < 0, y_e = c_e - (e_e - 1) > 0.$$

D.4 Dividend

In this section, we solve for the dividend loading on the three shocks. The dividend can be written as:

$$d_t = \frac{1}{1-\theta_w}[y_t - \theta_w(w_t - p_t - n_t)] - p_{ht}$$

$$= \frac{1}{1-\theta_w}\left[y_t - \theta_w\left(\frac{\varphi}{1-\alpha}y_t + \gamma(\hat{\alpha}_c c_{ct}^w + (1-\hat{\alpha}_c)(c_{et}^w + \delta_t) + \frac{1-\hat{\alpha}_c}{\phi}(c_{ct}^s - c_{et}^s - \delta_t) + \frac{1}{1-\alpha}y_t\right)\right]$$

$$-\frac{1-\hat{\alpha}_c}{\phi}(c_{ct}^e - c_{et}^e - \delta_t)$$

Apply the assumption that $\hat{\alpha}_c$ being close to 1, we can derive

$$d_\mu = \frac{1}{1-\theta_w}\left[\left(1 - \frac{(\varphi+1)\theta_w}{1-\alpha}\right)(c_\mu - e_\mu) - \theta_w\gamma(c_\mu - \frac{1}{\theta_w}e_\mu)\right] < 0$$

$$d_\delta = \left[1 - \theta_w\left(\frac{\varphi+1}{1-\alpha} + \gamma\right)\right]c_\delta + \left(-1 + \gamma + \frac{\varphi+1}{1-\alpha}\theta_w\right)e_\delta + \theta_w\left(\frac{1}{\phi} - \gamma\right)(1-\hat{\alpha}_c) + \frac{(1-\theta_w)(1-\hat{\alpha}_c)}{\phi}$$

$$> (1-\theta_w)\gamma c_\delta + \theta_w\left(\frac{1}{\phi} - \gamma\right)(1-\hat{\alpha}_c)\frac{(1-\theta_w)(1-\hat{\alpha}_c)}{\phi} > 0$$

The inequality comes from the fact that $\theta_w < \frac{(1-\alpha)(1-\gamma)}{1+\varphi}$. The sign of d_e is the same as d_δ.

D.5 The Stochastic Discount Factor

The stochastic discount factor is written as:

$$m_{t+1} = m_\mu(1-\rho_\mu)\mu_t + m_e(1-\rho_e)c_{et} + m_\delta(1-\rho_\delta)\delta_t - \lambda_\mu\sigma_\mu\varepsilon_{\mu,t+1} - \lambda_e\sigma_e\varepsilon_{e,t+1} - \lambda_\delta\sigma_\delta\varepsilon_{\delta,t+1}$$

In this model, only the shareholders' consumption matters for asset pricing:

$$m_{t+1} = -\gamma(c_{t+1}^s - c_t^s) = -\gamma[\hat{\alpha}_c(c_{c,t+1}^s - c_{c,t}^s) + (1-\hat{\alpha}_c)(c_{e,t+1}^s - c_{e,t}^s + \delta_{t+1} - \delta_t)]$$

It is straightforward to derive that:

$$\lambda_\mu = \gamma c_\mu < 0, \lambda_\delta = \gamma c_\delta + \gamma(1-\hat{\alpha}_c) > 0, \lambda_e = \gamma c_e + \gamma(1-\hat{\alpha}_c) > 0$$

$$m_\mu = \gamma c_\mu < 0, m_\delta = \gamma c_\delta + \gamma(1-\hat{\alpha}_c) > 0, m_e = \gamma c_e + \gamma(1-\hat{\alpha}_c) > 0.$$

D.6 Nominal SDF and Asset Prices

From the main text, we see that returns to core stocks, currencies, and commodities only depend on loadings of dividend and SDF on the three shocks. All derivations in the main text apply in the extended model. For bond returns, we need to derive the asset return loadings using the nominal SDF. We derive the loadings of nominal SDFs here as well, $E_t m^{\$}_{t+1} = E_t m_{t+1} - E_t \pi_{t+1} - E_t(p_{h,t+1} - p_{h,t})$.

$$m^{\$}_\mu = m_\mu(1-\rho_\mu) - \rho_\mu \pi_\mu + \frac{1-\hat{\alpha}_c}{\phi}(c_\mu - e_\mu)(1-\rho_\mu) < 0$$

$$m^{\$}_\delta = m_\delta(1-\rho_\delta) - \rho_\delta \pi_\delta + \frac{1-\hat{\alpha}_c}{\phi}(c_\delta - e_\delta - 1)(1-\rho_\delta)$$

$$= (1-\rho_\delta)\left[\gamma c_\delta + (1-\hat{\alpha}_c)\left(\gamma - \frac{1}{\phi}\right) - \rho_\delta \pi_\delta\right]$$

Since $-\gamma c_\delta + \left(\frac{1}{\phi} - \gamma\right)(1-\hat{\alpha}_c) = \frac{\phi_\pi - \rho_\delta}{1-\rho_\delta}\pi_\delta$, $m^{\$}_\delta = (1-\rho_\delta)\left(-\rho_\delta - \frac{\phi_\pi - \rho_\delta}{1-\rho_\delta}\right)\pi_\delta < 0$. Similarly, $m^{\$}_e < 0$.

Since the sign of real and nominal SDF loadings and dividend loadings are exactly the same as in the model presented in the main text, all asset return loadings are identical, too. We skip all the derivations of asset prices here for the extended model.