The Allocation of Socially Responsible Capital

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Abstract

Portfolio allocation decisions increasingly incorporate social values. The most common of these strategies are misguided. We develop a tractable framework in which commercial and social investors compete, and identify alternative strategies for social investors that result in higher social welfare and higher financial returns. From the enterprise perspective, increasing profitability can have a greater social impact than directly increasing social value creation. Whether investors and firms exhibit positive or negative assortative matching depends on the nature of social preferences. We present empirical evidence that socially-guided mutual funds allocate their capital inefficiently from the perspective of generating impact and financial returns.

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1 Introduction

The last several decades have seen an invigoration of investing in companies that rank favorably on metrics of social value, such as environmental stewardship, social responsibility, and good governance practices (collectively referred to as ESG). For instance, one quarter of assets under management by professional investors–$30 trillion– are now allocated with such considerations (US SIF Foundation, 2018). This large shift in investment strategies has the potential to dramatically alter the allocation of capital in the economy. In fact, many argue the entire purpose of this movement is to help reallocate resources to socially beneficial uses and away from socially harmful ones. Thus, it is centrally important to understand whether and how this style of investing generates its intended "impact." This paper develops a theoretical framework to explore how investing with social convictions results in the creation of social value and which investment strategies generate social value most efficiently.

We focus on strategies based entirely on selecting which assets to hold or avoid. The most common of such strategies in practice are constructed with attention to the financial returns and the social value of the companies included in an investor's portfolio. For example, ESG index funds attempt to track the returns of a benchmark index while maximizing some composite measure of the social good of the companies in the portfolio. Proponents of such "values-aligned" investing claim that they increase the valuation of (or equivalently decreases the cost of capital for) economic endeavors that contribute the most positively to society. This in turn shifts the set of projects that markets will finance towards those that create social value and away from those that destroy it.

We argue that the folk wisdom justifying "values-aligned" investing is misguided, and such investment strategies are an inefficient way to use asset allocation decisions to influence social value creation. Our framework builds on the insight that an investor's true contribution to social value is not reflected in the social value of the companies in their portfolio, but rather by the additional social value created relative to if the investor did not exist at all (e.g. Brest et al., 2016). The distinction between these perspectives is driven by the fact that many companies that have high social value could attract investors with a purely financial objective. Therefore, socially motivated investors who finance these companies may not be contributing to social value creation. In fact, their behavior could even result in social value destruction if it displaces investors unconstrained by social considerations into financing socially harmful projects. We formalize this critique in an equilibrium model of capital allocation and characterize its implications for the behavior of social investors. We further identify an alternative to "values-aligned" investment strategies that can generate more impact *and* achieve higher financial returns.

To understand the basic logic of why social investors who attend only to the social value of their portfolio companies might achieve sub-optimal outcomes, consider the following stylized example.

Suppose there are two investors each of whom holds one unit of capital. One *commercial investor* cares only about financial returns and the other *social investor* cares about both financial returns and social value. Suppose further that there are three enterprises, each of whom needs one unit of capital to operate:

- Firm *A* generates a 10% profit and 10 units of social value.
- Firm *B* generates a 8% profit and 5 units of social value.
- Firm *C* generates a 9% profit and 0 units of social value.

Investors finance companies at terms that ensure them a financial return no greater than the firm's total return on investment. A social investor who makes investment decisions based only on their own returns and the social value of the companies they finance would want to invest in Firm *A*. Such an investor would be willing to pay more for this opportunity than the commercial investor, leaving the commercial investor to finance Firm *C* and earn a 9% return. As a result, the social investor finances Firm *A*, offers capital at a cost of 9% (so as not to be undercut by the commercial investor), and 10 units of social value are created. If instead the social investor took a holistic view, they would appreciate that Firm *A* is profitable enough to attract the support of the commercial investor. The social investor might then want to invest in firm *B*. In this case the social investor would receive a financial return of 8% and 15 units of social value would be created.

This example highlights that social investors narrowly focused on the social value *attributable* to companies in their own portfolio are not effective at *generating* social value through their investment decisions.[1] We develop a framework to embed this logic in a competitive financial market, in which many commercial investors and social investors coexist and firms' costs of capital are determined in equilibrium. To highlight the nuances arising from the two approaches to social investing in the previous example, we introduce two types of social investors. *Values-aligned social investors* form portfolios based on the financial returns and social welfare generated by the companies they support. *Impact-aligned social investors* are similar, but consider implications of their investment decisions for total social welfare.

Beyond admitting a tractable analysis of equilibrium behavior, our model yields several normative implications for social investors and entrepreneurs. First, we identify improvements to the investment strategy of values-aligned social investors. Any capital that values-aligned investors deploy to fund profitable but also impactful projects that could have been commercially financed should be redeployed to projects with lower profitability (and potentially lower social value). Perhaps surprisingly, this can increase both the social impact *and* financial return to this capital. Why? As the result of

[1] In fact, notice in this example that the social investor's choice of Firm A results in *zero* additional social value creation relative to an economy in which neither investor had preferences for social value.

values-aligned investors competing with each other to own shares in certain projects, they are both displacing commercial investors *and* transferring excessive value to entrepreneurs (or a firm's existing owners). Any financial concession made to own a firm that could have attracted commercial investment does not serve to expand the set of socially valuable projects that are economically viable. This generates scope to put a financial concession to more effective use from the perspective of generating impact. Instead, investing directly in projects that are less profitable but more impactful than what a displaced commercial investor would have chosen can thus result in higher social value creation *and* higher financial returns to the social investor.

Our baseline model considers an environment where all firms have a fixed scale. In this case, impact-aligned social investors support firms that could not attract any commercial financing. This is not a realistic option for small socially conscious investors, who are typically limited to making investments through established capital markets. We extend our model to investigate the implications of socially responsible investing when firms have an intensive margin of scale and show that in this case there is scope for creating impact by investing in commercially viable firms. We illuminate the role of "blended finance"–issuing claims at different prices for different investors—in maximizing a social investor's impact. Securities resembling green bonds emerge in equilibrium to allow impact-aligned social investors to provide low-cost financing for socially valuable projects without displacing a firm's existing commercial investors. In contrast, there is no role for blended finance when social investors are values-aligned. Further, our analysis highlights that green bonds are only impactful if they are used to finance investments that are not only "green", but also are not profitable at the commercial cost of capital.

Our framework also has implications for evaluating the social impact of a firm, sometimes called its *enterprise impact* (Brest et al., 2016). Enterprise impact depends not only on the amount of capital used by the enterprise, but also on the *type* of capital used by the enterprise. All else equal, enterprises that attract the capital of socially minded investors have a lower contribution to social welfare than those that attract the capital of purely commercial investors. Holding fixed the social value created by a firm, it can raise its enterprise impact by reducing its dependence on social capital, freeing social capital to fund another enterprise that is unable to obtain commercial financing. The more profitable a firm, the less likely it is to rely on the scarce, socially valuable capital of impact-aligned investors, as impact-aligned investors avoid supporting firms that could attract commercial capital. Our framework thus provides a new connection between the profitability of an enterprise and its contribution to social welfare.

Finally, we study the equilibrium matching of investors to entrepreneurs when social investors exhibit varying degrees of altruism. When social investors are values-aligned we identify a familiar result: social investors and entrepreneurs exhibit positive assortative matching, in that investors who

care more about social welfare match to entrepreneurs that create more social value. However when social investors are impact-aligned, this result reverses. Holding fixed the level of an entrepreneur's profitability, social investors who value social welfare more highly match to entrepreneurs who create less social value. This can be viewed as an extension of the core logic of the paper; impact-aligned investors do not want to displace investors with less concern for social value, whether that be commercial investors, or other impact-aligned investors with a lower degree of altruism. More generally, this result arises from the fact that social investors have interdependent utility, in that they internalize the social value created by all firms that receive financing.

How important are our findings in practice? This depends on the extent to which socially-minded investors adopt values-aligned versus impact-aligned strategies. We present empirical evidence that the investment strategies adopted by "sustainable" equity mutual funds strongly mirror those of the values-aligned investors in our model. In particular, we compare holdings of funds incorporating sustainability objectives to the broader universe of mutual funds, and find no evidence that these sustainable funds have reduced holdings in highly profitable high social-value companies that could have obtained commercial capital. Our results therefore suggest investors in these funds could improve both financial returns and contribution to social value.

This paper contributes to the literature on investing with social preferences. A number of theoretical papers study financing environments where social and commercial investors coexist, and ask how social investors should behave to maximize their impact. Oehmke and Opp (2019) Landier and Lovo (2020), and Broccardo et al. (2020) study activist social investors who aim to resolve a moral hazard problem among entrepreneurs.[2] In contrast to these papers, we study passive investors in a complete information environment, whose goal is to enable new projects by offering cheaper capital to firms with socially valuable projects.

Heinkel et al. (2001), Pedersen et al. (2019) , Pastor et al. (2020), and Goldstein et al. (2021) examine markets in which passive social and commercial investors co-exist and study equilibrium asset allocation and prices. In contrast to these papers, we study a model without uncertainty, and focus our analysis on the behavior of impact-aligned social investors who aim to maximize social welfare rather than the impact of their own portfolio.[3] Moisson (2020) has a related ambition, contrasting the behavior of consequentialist social investors with that of investors who hold other moral criteria.

Several papers analyze the behavior of individual firms and their prosocial investors. Focusing on

[2]Thakor and Quinn (2020) also studies how the presence of social and commercial investors interacts with the firm's moral hazard problem.

[3]Pastor et al. (2020) considers an extension in which investors incorporate the social value of all firms into their objective function, and hence correspond to our impact-aligned investors. But the analysis assumes that individual investors cannot meaningfully influence the economy and hence their preferences for financial return and the social value of firms within their own portfolio fully determine their investment decisions.

the single investor case, Chowdhry et al. (2019) and Roth (2020) analyze when a socially minded investor can have more impact through an investment in a social enterprise than they can through a grant. Hart and Zingales (2017) fleshes out several cases for a stakeholder view of the firm, and Morgan and Tumlinson (2019) and Friedman and Heinle (2021) highlight collective action concerns among firms' prosocial shareholders. Dewatripont and Tirole (2020) study how competition affects the degree to which firms' behaviors reflect the ethical concerns of their stakeholders.

Our study also relates to the economic literature on altruistic motives. Andreoni (1990) highlight the distinction between "pure altruists," who derive utility from social welfare, and "impure altruists," who derive utility, or "warm glow" from having directly improved social welfare. In this light, our impact-aligned social investors can be understood as pure altruists, and our values-aligned social investors can be understood as impure altruists. We contribute to this literature by embedding these various altruistic motives into a model of capital allocation.

Our analysis bears a technical resemblance to assignment matching models, commonly employed in trade and labor economics (e.g. Roy, 1951, Becker, 1973, Sattinger, 1979, Costinot and Vogel, 2010). We contribute to this literature by providing a model in which agents sort along two dimensions of heterogeneity, as in Gola (2020), and by studying an environment where one side of the market has interdependent utility in the sense that they care not only about their own match, but also the matches of others. As discussed in Section 5, we show that this latter feature can partially reverse the classic result of positive assortative matching.

Finally, a large empirical literature examines the strategies of social investors[4] and the consequences of social preferences on financial returns[5], asset pricing, and access to capital[6]. Our paper provides a normative theory for how investors should behave if they care directly about their social impact in addition to their financial return. A smaller literature examines the relationship between ESG scores and firm characteristics.[7] Our paper can be understood as providing guidance on how to construct ESG scores to guide investors seeking to maximize their impact. Further, we build on the empirical literature by providing an analysis of the holdings of investors with social preferences. In conjunction with our model, the evidence suggests that sustainable mutual funds behave as if they have values-aligned, rather than impact-aligned preferences.

The rest of the paper proceeds as follows. Section 2 outlines the model for the case where entrepreneurs have binary projects and Section 3 presents the results in this setting. In Section 4 we

[4]e.g. Dimson et al. (2015), Krueger et al. (2019), Hartzmark and Sussman (2019), Dimson et al. (2020), Geczy et al. (2020), Matos et al. (2021)

[5]e.g. Barber et al. (2019), Engle et al. (2019), Pastor and Vorsatz (2020), Cole et al. (2020), Hoepner et al. (2021), Jeffers et al. (2021)

[6]e.g. Teoh et al. (1999), Varma and Nofsinger (2012), Kacperczyk (2009), Chava (2011), Ilhan et al. (2020), Koijen et al. (2020)

[7]e.g. Karolyi et al. (2020), Elmalt et al. (2021)

consider the case where entrepreneurs have a non-trivial intensive margin of scale. In Section 5 we consider investors with varying degrees of concern for social welfare. Section 6 presents empirical evidence. Section 7 concludes.

2 Baseline Model

Players, Technology, and Contracts

There is a finite set E of entrepreneurs. Each one is endowed with a project that requires one unit of capital. If entrepreneur i receives the requisite capital, their project returns $\pi_i \in [0, \hat{\pi}]$ profit and $w_i \in [-\hat{w}, \hat{w}]$ "social value," where π_i and w_i represent the private and social return of the project respectively.[8] We assume that the features of each project are perfectly observable to all players.

There is a finite set S of social investors, each of whom allocates one unit of capital.[9] There is also a market for commercial capital that inelastically supplies financing to all firms at required rate of return of r^C.

A contract between some investor and an entrepreneur i specifies the transfer of one unit of capital from the investor to the entrepreneur in exchange for financial return r_i on their invested capital. The entrepreneur receives a share of profits $\pi_i - r_i$, and w_i social value is created. We will sometimes refer to r_i as the price or cost of capital offered to an entrepreneur. Because we are studying a complete information environment without contracting frictions, this contract can be understood as either debt or equity. In addition to being able to finance the entrepreneurs in E, social investors can also allocate their capital to a "social value-neutral" asset with financial return r^C profit and 0 social value.[10]

Preferences

We index investors and entrepreneurs such that investor i matches with entrepreneur i. Each entrepreneur's utility is their share of the profit, $(\pi_i - r_i)$. We will separately examine two classes of social investors.

Values-aligned social investors make investment decisions based on financial returns they receive

[8] We assume that w_i encompasses the full social return of the project, including the private return π_i, as well as any consumer and employee surplus and externalities arising from the project.

[9] We discuss how the analysis can be extended to a model with a continuum of investors and projects in Appendix Section A.6.

[10] The results would be nearly unchanged if social investors did not have access to such a value-neutral asset.

and the social value created by the entrepreneur they have financed. That is, their utility is

$$(r_i + \theta w_i),$$

where θ represents the strength of investor i's social preference.

Impact-aligned social investors receive utility from their income and from the total social value created by all entrepreneurs who receive financing. That is, their utility is

$$r_i + \theta \sum_{j \in \bar{E}} w_j = (r_i + \theta w_i) + \theta \sum_{j \in \bar{E} \setminus i} w_j,$$

where $\bar{E}$ is the set of entrepreneurs who receive financing.[11] We can observe that the difference between the utility functions of values-aligned and impact-aligned social investors is that impact-aligned social investors derive utility equally from all social output regardless of who financed it.[12] The implication of this difference is that impact-aligned investors internalize the consequences of their actions on total social welfare, while values-aligned investors only consider the social value of the firm they finance.

Values-aligned investors do not fully internalize the implications of their investment decision on social welfare. This does not imply that the preferences of values-aligned investors are incorrect. Values-aligned investors may derive intrinsic utility from owning firms that create social value, similar to the conception of warm-glow altruists in Andreoni (1990). In such a case, the analysis to follow should be understood as exploring the positive implications of these two modes of investment behavior. Alternatively, values-aligned preferences may represent the *behavior* of socially conscious investors, while impact-aligned social preferences may more faithfully represent the *intentions* of socially-conscious investors to affect social change. Under this interpretation, our analysis of the behavior of impact-aligned social investors offers normative guidance to real-world investors with social preferences.

[11] We note that as w_i includes r_i, both preferences above "double count" r_i. The values-aligned preferences can be rewritten as $r_i + \tilde{\theta}(w_i - r_i)$ and the impact-aligned preferences can be rewritten as $r_i + \tilde{\theta}\left(w_i - r_i + \sum_{j \in \bar{E} \setminus i} w_j\right)$, where $\tilde{\theta} = \frac{\theta}{1+\theta}$. Hence the two preferences can be equivalently stated, up to a re-normalization, without double counting r_i.

[12] The distinction between values-aligned and impact-aligned investors mirrors that of the "narrow mandate" and "broad mandate" of Oehmke and Opp (2019).

Timing of Actions

First, each social investor offers a contract to an entrepreneur.[13] Simultaneously, all firms receive an offer for commercial financing at rate r^C. Entrepreneurs then choose at most one contract to accept, and payoffs are realized.

Equilibrium

The solution concept is pure-strategy Subgame Perfect Equilibrium. All investors choose contracts that are mutual best responses. Among other things, this implies that no entrepreneur ever receives offers from more than one social investor. Therefore we maintain the convention that entrepreneur i matches with investor i.

Social Welfare

Our measure of social welfare is $W = \sum_{i \in \bar{E}} w_i$, where $\bar{E}$ is the set of entrepreneurs that receive financing. Our interpretation is that w_i is the total social value created by firm i if it receives financing, including the value to the firm's owners.[14] Impact-aligned social investors can therefore be understood to be maximizing a modified variant of social welfare that increases the weight placed on their own consumption. Also note that, consistent with Hart and Zingales (2017) and Broccardo et al. (2020), our measure of welfare does not include the "altruistic" utility that social investors derive from the creation of social value w_i.

3 Analysis of Baseline Model

To understand the behavior of values-aligned and impact-aligned social investors we first characterize the equilibrium of the model in which all investors are either values-aligned or impact-aligned. In Section 3.3 we present our main results in the model in which both types of social investors coexist.

[13] For technical convenience, we allow social investors to offer more than one contract. In the event that multiple contracts are accepted, one is chosen uniformly at random to be implemented.

[14] Under this interpretation, the value accruing to the firm's owners is determined independently of how ownership is divided, i.e. the welfare weights placed on entrepreneurs and investors are the same.

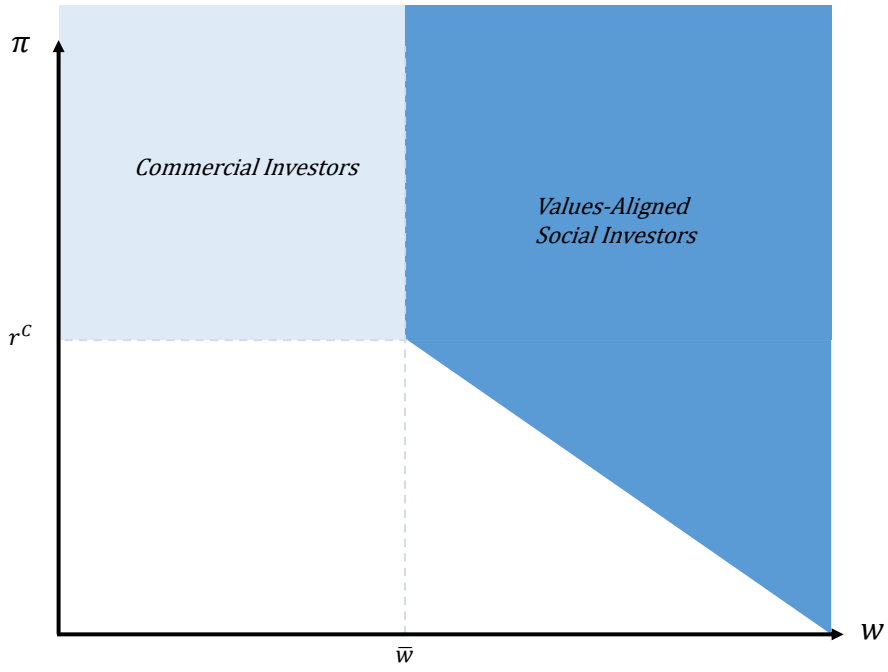

Figure 1: Equilibrium investment with values-aligned social investors

3.1 Values-Aligned Social Investors

We begin by characterizing the equilibrium of the model where all social investors are values-aligned. For any two entrepreneurs i and j who are both supported by a social investor, their costs of capital satisfy $r_i + \theta w_i = r_j + \theta w_j$.[15] And for an entrepreneur i supported by a social investor and an entrepreneur k who is not, prices of capital must satisfy $r_i + \theta w_i \geq r_k + \theta w_k$.[16] In equilibrium there exists a cutoff $\bar{w}$ such that commercial investors only support projects with social value below some $\bar{w}$ and profits above r^C. Competition between social investors pins down their utility to $r^C + \theta\bar{w}$. This implies social investor i receives financial return of $r_i = r^C - \theta(w_i - \bar{w})$. Thus, they are willing to pay (in terms of reduced financial return) for projects that generate high social value.

The equilibrium investment allocation is depicted in Figure 1. The graph shows the space of available investment opportunities, parameterized by π and w, and the shaded regions show the sets of entrepreneurs financed by different types of investors. Entrepreneurs in the unshaded region are unfinanced in equilibrium. A formal characterization of equilibrium investment allocations is relegated to the Appendix Section A.1.

[15] The preceding equality holds so long as costs of capital are positive. In equilibrium a social investor may provide funding in exchange for zero share of the proceeds ($r = 0$) if the project has sufficiently high social impact (akin to philanthropy). In such a case, the above equality need not hold.

[16] We adopt the convention that an entrepreneur i who accepts no offers for financing has a cost of capital $r_i = \pi_i$.

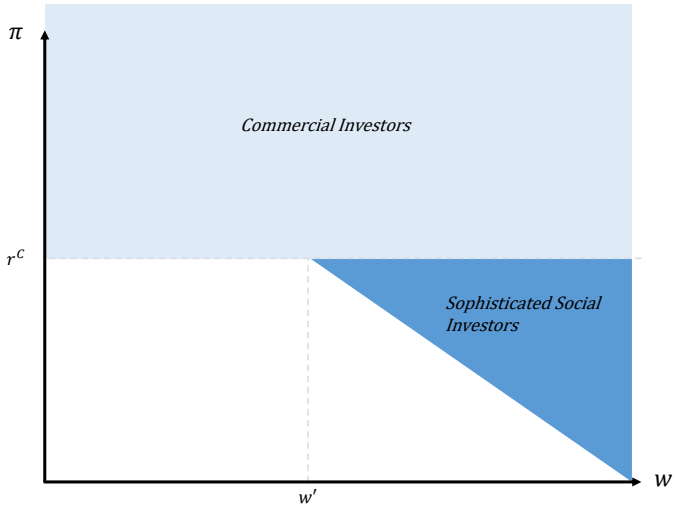

Figure 2: Equilibrium investment with impact-aligned social investors

3.2 Impact-Aligned Social Investors

Next we analyze the equilibrium allocation of capital when social investors are all impact-aligned. The equilibrium allocation is depicted in Figure 2.

In equilibrium, social investors expect that entrepreneurs with profits above r^C can receive financing from a commercial investor. Therefore, impact-aligned social investors recognize that by supporting an entrepreneur i with profits $\pi_i \geq r^C$, their marginal contribution to social welfare is not w_i, but rather 0, as the firm would receive financing independently of the social investor's offer.[17]

Further, among the set of entrepreneurs that cannot attract commercial financing —i.e. $\pi_i < r^C$— prices are such that impact-aligned investors extract their full profits. This is so because impact-aligned social investors recognize that they cannot contribute to social welfare by undercutting another social investor, and hence they would always prefer to invest in the value-neutral outside option asset returning r^C, rather than undercutting an investor who supports a firm with $\pi_i < r^C$.[18]

Therefore, in equilibrium impact-aligned social investors support the firms i who have the highest $\pi_i + \theta w_i$, amongst those that generate profits $\pi_i < r^C$ (so that they would not receive commercial financing). This is depicted in Figure 2. In Appendix Section A.2, we formally characterize the allocation of impact-aligned social investor's capital.

[17] In a full general-equilibrium model, the social investor's contribution to social welfare from supporting a commercially viable firm would be the social value of marginally expanding the set of commercially supported firms. We normalize this value to 0.

[18] Our analysis assumes that investors make take it or leave it offers; thus the fact that no impact-aligned investor seeks to undercut another implies that impact-aligned investors extract the full surplus (i.e. profit) of their investment. The analysis would be similar if bargaining power were such that entrepreneurs extracted some of the surplus as well.

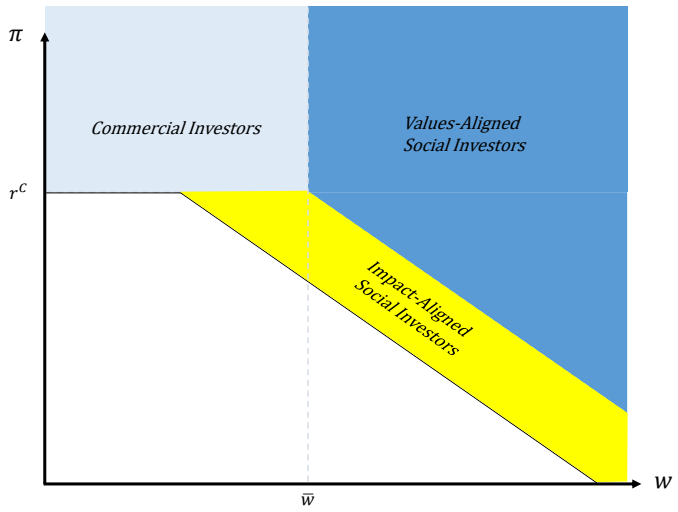

Figure 3: Equilibrium investment with both types of social investors

3.3 Main Results

In this section we discuss a number of normative results about social investing, in a market in which both impact-aligned and values-aligned social investors coexist. In particular, we demonstrate that there are investment strategies that deliver higher financial returns and create more social value than those employed by values-aligned social investors. And we draw a new link between the profitability of a firm and the firm's social value.

First we characterize the equilibrium in the market with both types of social investors.

Equilibrium Structure with Both Types of Social Investors

Let there be a mass S_V of values-aligned social investors, and S_I impact-aligned social investors.

There is no longer a unique equilibrium in this market. In Appendix Section A.3 we characterize the full set of equilibria. In this section we describe the investor-optimal equilibrium – i.e. the equilibrium that maximizes the sum of investor utilities – which is also the welfare-optimal equilibrium, depicted in Figure 3. All of the propositions in Section 3.3 apply to all equilibria.

As in Section 3.1 there is some $\bar{w}$ such that values-aligned social investors only support entrepreneurs with social value greater than $\bar{w}$ and commercial investors only support entrepreneurs with social value less than $\bar{w}$. Entrepreneurs with profits higher than r^C and social value less than $\bar{w}$ are supported by commercial investors.

In the investor-optimal equilibrium impact-aligned social investors do not compete with either commercial investors or values-aligned social investors, and instead support the set of entrepreneurs i who maximize $\pi_i + \theta w_i$ amongst those who could not attract financing from other investors. We defer formal characterization of this equilibrium to Appendix A.3. For the remainder of our analysis we focus on the investor-optimal equilibrium to illuminate the model's comparative statics.

Reallocating Values-Aligned Social Capital to Improve Social Welfare and Financial Returns

In this section we consider two thought exercises. First, holding fixed the equilibrium behavior of all other investors, we consider the possibility of reallocating the investment of a single values-aligned social investor. We demonstrate that any values-aligned social investor who supports a firm with $\pi_i \geq r^C$ and who earns a financial return of $r_i < r^C$ could reallocate their capital to increase total social welfare *and* increase their financial return. In this sense values-aligned investors leave both money and impact on the table. We then consider the possibility of converting a values-aligned social investor into an impact-aligned social investor, and show that this always leads to an increase in social welfare and sometimes (but not always) leads to an increase in the investor's financial return.

The logic of the result follows from the fact that values-aligned investors place intrinsic value on owning socially valuable firms. Values-aligned investors compete not only with commercial investors, but also with one another to own socially valuable firms, thereby bidding up their prices. While some of this competition makes new socially valuable firms economically viable, some of it results in rents for entrepreneurs with socially valuable firms that would have been viable at the commercial cost of capital. Therefore, values-aligned investors make a financial concession relative to commercial investors, and some of that concession is wasted from the perspective of social value creation. This creates scope to reallocate values-aligned capital in a way that reduces their financial concession and increases aggregate social value.

Proposition 1. *Consider any values-aligned social investor i that supports a firm with $\pi_i > r^C$ and earns a return $r_i < r^C$ in equilibrium. If the distribution of firms is sufficiently dense, there exists an unfinanced firm j with profits $\pi_j > r_i$, such that if the values-aligned social investor i were to deviate and offer firm j financing at cost π_j, total social welfare would increase as would investor i's financial return.*

In the proof of Proposition 1 we formalize the notion that the distribution of entrepreneurs is *sufficiently dense*. Intuitively, it guarantees that for any combination of π and w, there is an entrepreneur with profits π_i near π and social value w_i near w. Proposition 1 can be understood with reference to Figure 4. Fix any values-aligned social investor i that supports a firm i with profits $\pi_i \geq r^C$ and who earns financial return $r_i < r^C$ (generically this holds for all values-aligned investors who support firms

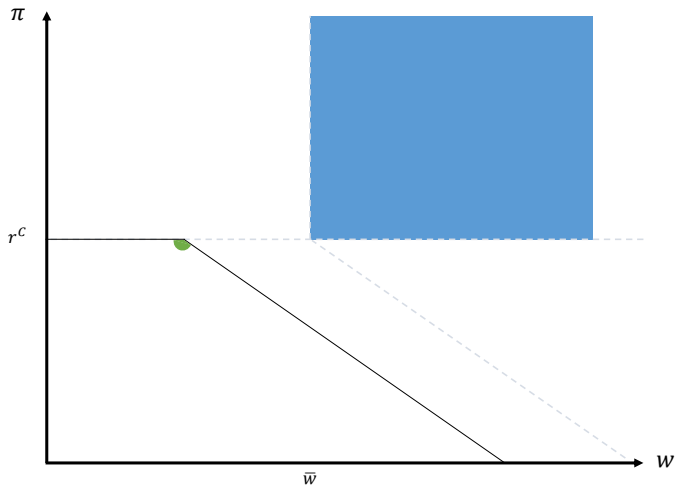

Figure 4: Reallocating Values-aligned Social Capital Out of Equilibrium

with $\pi_i \geq r^C$). These investors support the firms highlighted in blue. And consider among the set of unfinanced firms some firm j with profits $\pi_j > r_i$ and with social value $w_j > 0$.[19] This firm is guaranteed to exist by the assumption that the distribution of firms is dense. One such firm is highlighted in green.

The contribution to social welfare of the equilibrium investment for investor i is 0, regardless of the social value w_i of firm i, as investor i is merely displacing commercial investment. Firm j creates less social value than any firm in the blue region of the diagram, but by reallocating investor i's capital to firm j social welfare increases, as firm j was previously unfinanced.

Further, by offering firm j a cost of capital $r_j = \pi_j$, investor i can earn higher financial return as well. As with social value, firm j earns lower profits than any firm in the blue region of the diagram. But because $\pi_j > r_i$, the subsidy required to make firm j NPV positive is smaller than the financial concession (relative to the commercial rate of return) that investor i made to own firm i. Critically, values-aligned social investors compete down the price of capital of firms with large contributions to social value even when these firms could have attracted commercial financing. The financial compromise made by values-aligned investors to support such firms results in a transfer of wealth to the entrepreneur rather than expanding the pool of socially valuable firms. In contrast, the financial compromise made to support a firm that could not attract commercial financing goes entirely toward expanding the pool of socially valuable firms rather than transferring rents to entrepreneurs whose projects would anyway have been feasible.

[19] Or, more generally, we identify a firm j with profits $\pi_j > r_i$ and with social value w_j greater than the social value of the marginal firm receiving commercial support.

We note that the proof of Proposition 1 relies on allowing the reformed values-aligned investor to extract the full profit of the previously unfinanced firm highlighted in green in Figure 4. However this feature of the model is not necessary for the result. The critical ingredient is that values-aligned investors bid up the price of some firms in a way that does not contribute to aggregate social value (i.e. by competing up the price of socially valuable projects that would have occurred even at the commercial cost of capital). So long as there is a way to reallocate a portion of that financial concession in a way that *does* contribute to social value (e.g. by donating it to charity), then the reformed values-aligned investor can reduce the total financial concession and allocate the remainder in a way that creates social value – this increases both impact and financial return.

Proposition 1 demonstrates that values-aligned social investors leave both impact and money on the table in the sense that, relative to the firms these investors support, there are unfinanced firms that could deliver higher financial returns and increase social welfare. However, while it is straightforward to show that converting values-aligned social investors to impact-aligned social investors would result in higher total social welfare, in general we cannot guarantee that this conversion would lead investors to earn higher financial returns. The simple reason is that once values-aligned investors have been converted to impact-aligned social investors, while they would prefer to finance firms in the green region of Figure 4 relative to any firm in the blue region, there may be yet another firm they prefer to firms in the green region that contributes more to social welfare but has lower financial return. Nevertheless, we can demonstrate the following result.

Proposition 2. *There may exist a set of values-aligned social investors such that were they to be converted to impact-aligned social investors they would earn higher returns. Moreover total social welfare would increase.*

Finally, we note that while converting these values-aligned social investors to impact-aligned social investors increases their profits and total social welfare, it does not increase their utility judged according to the utility function of values-aligned social investors. Nevertheless, Proposition 2 offers encouraging news about the prospect of converting values-aligned social investors to impact-aligned social investors in practice. A substantial amount of effort has gone into investigating the hypothesis that ESG investing can increase impact and financial returns (e.g. Eccles et al., 2014), suggesting investors are sensitive to the financial and social implications of values investing. Our model demonstrates that relative to conventional ESG strategies there is room for improvement in both dimensions.

Enterprise Impact

How should one judge the contribution to social welfare of a particular entrepreneur, sometimes referred to as *enterprise impact* (e.g. Brest et al., 2016)? On first pass it might seem natural for w_i to

be the measure of enterprise impact. However, we argue that a firm's enterprise impact should also account for the social value of the capital it employs.

Let $W(S_V, S_I)$ be the total social value created in equilibrium given masses of investors S_V, and S_I. Define v_{S_V} to be the increase in social value corresponding to adding one additional values-aligned social investor, and v_{S_I} to be the increase in social value corresponding to adding one additional impact-aligned social investor. The social value of commercial capital, v_C is normalized to 0. It is straightforward to show that $0 < v_{S_V} < v_{S_I}$.

We define the enterprise impact of firm i to be $e_i \equiv w_i - v_i$ where v_i is the social value of capital attributable to the investor who supports entrepreneur i in equilibrium. We define the enterprise impact to be 0 for firms that do not receive financing.

This definition of enterprise impact might have practical value for socially motivated investors aiming to quantify the social value of a particular enterprise. Frontier efforts in the impact investing industry often attempt to account for the social value created by the enterprise and the amount of capital employed by the enterprise, such as in the *impact multiple of money* method (Addy et al., 2019). Our analysis highlights that it is also critical to account for the composition of social capital versus commercial capital raised by an enterprise in judging its impact.

This definition of enterprise impact also highlights an alignment between the enterprise impact and profitability of a firm. Firms can increase their enterprise impact by increasing their profitability *even holding fixed their social value* w_i. Increasing the profitability of the firm makes it more likely to attract commercial capital, freeing up capital that is willing to accept lower returns to fund higher social value endeavors. In particular, we have the following result.

Proposition 3. *Suppose firm i attracts financing from an impact-aligned social investor in equilibrium. Increasing its profits π_i while holding fixed its social value w_i increases its enterprise impact e_i and total social value created in equilibrium.*

Proposition 3 states that making a firm more profitable increases its enterprise impact *even holding its social value* w_i *fixed.* Importantly, this result is not driven by an assumption that a firm's profitability and its social value are correlated. Instead, this result is driven by the observation that once a firm becomes profitable enough to attract commercial financing, impact-aligned social investors will step aside, freeing up their capital to support another socially valuable firm. Therefore, more profitable firms use less socially valuable capital, and have higher enterprise impact. Note that this phenomenon does not hold for firms supported by values-aligned social investors, as values-aligned social investors pay no regard to whether a firm could attract commercial capital in their absence.

4 Impact on the Intensive Margin

So far we have assumed that every firm has a single project, which is completed if and only if it raises a unit of capital. Within this setting, we demonstrated that the impact-aligned investing strategy can outperform the values-aligned investment strategy on both total social value creation and financial returns. However, the impact-aligned approach required that investors allocate their capital to firms that are not commercially viable. In reality, this would effectively relegate impact-aligned social investors to private capital markets, which is likely infeasible for small investors.[20] In this section we consider a variant of the model in which firms have continuous, concave production functions and demonstrate that impact-aligned social investors *can* have impact by inducing commercially viable firms to change their scale of operation. Therefore, there may be room for impact-aligned social investors to induce change in public markets.

The economic logic in the continuous-project case is largely the same as in the binary-project case. Values-aligned investors bid up the prices of firms with high average social value while impact-aligned investors target their subsidies to firms with socially valuable but marginally unprofitable projects. Relative to the binary-project case, two new insights emerge. First, impact-aligned investors maximize their impact by leveraging commercial capital. In equilibrium a capital structure emerges in which commercial and impact-aligned investors co-invest in the same firms at different terms, referred to as "blended finance" in the impact investment community.[21] In contrast, values-aligned investors never support the same firms as commercial investors. Second, impact-aligned investors utilize a security that resembles green bonds as a mechanism to subsidize firms without displacing commercial investors.

We finally show that our main results from Section 3 have natural analogues in the continuous-project case.

[20] Jeffers et al. (2021) analyze the financial returns of private market investment funds with a mandate to generate social impact.

[21] The term "blended finance" is increasingly prevalent amongst impact investors in practice. The International Finance Corporation of the World Bank defines blended finance as "the use of relatively small amounts of concessional donor funds to mitigate specific investment risks and help rebalance risk-reward profiles of pioneering investments that are unable to proceed on strictly commercial terms. Concessional funds are structured as co-investments, with an expectation of reflows for future investments or other uses."

4.1 Model

Agents, Technology, and Contracts

The model in Section 2 is now modified in the following ways. First, rather than assuming that firms have projects of binary scale, we now assume that each firm i can absorb any positive mass of capital k. Firm i then produces $\pi_i(k)$ profit and $w_i(k)$ social value. Both functions are increasing, concave, and continuously differentiable, although the case where $w(\cdot)$ is at some points decreasing could be easily accommodated. We maintain the assumption that there is a finite set of social investors denoted by S, each of whom owns one unit of capital. However, now capital is divisible so that in principle one investor could support several firms. There is a market for commercial capital that inelastically supplies financing to all firms at rate r^C. Social investors can also allocate their capital to a "social value-neutral" asset with financial return r^C profit and 0 social value.[22]

A contract between an investor and entrepreneur now specifies not only the transfer of capital from the investor to the entrepreneur at a cost of $r \geq 0$, but also a minimum-scale contingency k, discussed below. A contract is therefore represented by $\langle r, k \rangle$, where r represents the cost of 1 unit of capital. The minimum-scale contingency can also be left unspecified, represented by $\emptyset$.

Preferences

Let x_i^j be the amount of capital that investor i allocates to firm j in equilibrium, r_i^j be the cost of capital that investor i charges firm j and $\bar{k}_j$ be the mass of capital raised by firm j. A values-aligned social investor i's utility is represented by

$$\sum_j x_i^j \left(r_i^j + \theta \frac{w_j\left(\bar{k}_j\right)}{\bar{k}_j} \right).$$

That is, values-aligned social investors care about their total financial return and social value of the firm they support, weighted by their ownership share.

In contrast, an impact-aligned social investor i's utility is represented by

$$\sum_j x_i^j r_i^j + \theta \sum_j w_j\left(\bar{k}_j\right)$$

That is, impact-aligned social investors care about their financial return and the total social value created by all firms, regardless of who supports them.

[22] The results would be nearly unchanged if social investors did not have access to a social value-neutral asset.

Entrepreneur j has preferences represented by $\pi_j\left(\bar{k}_j\right) - r^j$, where $r^j \equiv \sum_i x_i^j r_i^j$, and the sum is taken over all contracts the entrepreneur accepts from investors i. That is, entrepreneurs maximize firm profit net of the cost of external capital.

Timing

First, all social investors offer a contract to an entrepreneur. Simultaneously, entrepreneurs receive offers from the commercial market for an arbitrary amount of capital at rate r^C. Second, entrepreneurs accept any number of such contracts and operate at scale $\bar{k}_i$, where $\bar{k}_i$ represents the mass of capital they have accepted. Entrepreneurs may only accept a contract that specifies a minimum-scale contingency k if they operate at a scale $\bar{k}_i \geq k$.[23]

Finally, we maintain the solution concept is pure-strategy Subgame Perfect Equilibrium.

Discussion of Minimum-Scale Contingency

The minimum-scale contingency gives investors the ability to influence a firm's scale on the margin. Without it, if an investor offered a firm cheaper capital than it could attract on the commercial market, the firm may merely accept that capital as a substitute for more expensive capital without changing its scale. A minimum-scale contingency was implicit in the binary-project model because by definition, if an investor offered subsidized capital to a firm that could not attract commercial financing, it would necessarily operate at a larger scale. Unlike in the binary-project model, in the continuous case social investors may desire to change the scale of a firm that could attract a non-zero amount of commercial capital, and the minimum-scale contingency offers them a route to do so. Minimum-scale contingencies resemble green bonds in that they require a firm undertake a specific project (or reach a specific scale) in exchange for financing.

4.2 Equilibrium Structure

For simplicity we focus separately on the case when all social investors are values-aligned and when all social investors are impact-aligned.

[23]For technical convenience, we allow social investors to offer more than one contract. In the event that multiple contracts are accepted, one is chosen uniformly at random to be implemented.

Values-Aligned Social Investors

The equilibrium when social investors are values-aligned works in much the same way as the binary-project case. Among all firms that receive financing, equilibrium sorting is such that there is a cutoff $\bar{w}$ for which social investors finance all firms with average social value $\frac{w_i(\bar{k}_i)}{\bar{k}_i} > \bar{w}$, and commercial investors finance the rest. Firms financed by the commercial market have a cost of capital r^C, and firms i and j financed by social investors have a cost of capital pinned down by the following indifference condition among social investors

$$r_i + \theta \frac{w_i\left(\bar{k}_i\right)}{\bar{k}_i} = r_j + \theta \frac{w_j\left(\bar{k}_j\right)}{\bar{k}_j}. \tag{1}$$

which implies that firms with higher average social value have lower costs of capital.

On the firm's side, firms choose their scale in one of two ways. Firms that are unconstrained by a minimum-scale contingency choose their scale $\bar{k}_i$ to maximize $\pi_i(k) - r_i k$, so that $\bar{k}_i$ solves

$$\pi_i'\left(\bar{k}_i\right) = r_i. \tag{2}$$

Firms that are constrained by a minimum-scale contingency need not set their marginal profit equal to their cost of capital. Specifically, let a firm's commercial scale k_i^C solves $\pi_i'\left(k_i^C\right) = r^C$, and define its commercial share of profits to be $q_i^C \equiv \pi_i\left(k_i^C\right) - r^C k_i^C$. A firm constrained by a minimum-scale contingency $\bar{k}_i$ need only satisfy

$$\pi_i\left(\bar{k}_i\right) - r_i \bar{k}_i \geq q_i^C \tag{3}$$

Equilibrium is determined by a set of costs of financing $\{r_i\}$ that satisfies the above equation 1, and a set of firm scales $\left\{\bar{k}_i\right\}$ each of which either satisfies Equation 2 or 3. We note that there exists an equilibrium where no investor utilizes scale-contingencies, and Equation 2 determines the scale of all firms.

Finally, we note that across all equilibria, there is no co-investment between commercial investors and social investors within any firm that receives a subsidy relative to the commercial cost of capital.

Lemma 1. *Firms that receive a cost of capital $r_i < r^C$ from any values-aligned social investor are financed wholly by values-aligned social investors.*

Social and commercial investors disagree on the relative value of companies with the same profits but different contributions to social value, so there is no equilibrium price at which both sets of investors would be happy to finance the same investment.[24] While this extreme separation would not

[24]The one exception is the firm for which $\frac{w_i(\bar{k}_i)}{\bar{k}_i} = \bar{w}$. This firm may be financed by both commercial and social investors

arise in a model with, for example, diversification motives, it illustrates an important point. Disagreement about the value of a company among investors implies that to change the scale of the company requires displacing some of its existing investors. This idea is closely related to the observations of Heinkel et al. (2001) and Broccardo et al. (2020), that commercial investors will partially "undo" the actions of social investors, insofar as social investors may partially crowd out commercial investors in the firms they support. We will see in the following section that impact-aligned social investors do not displace commercial investors, and co-investment does occur in equilibrium.

Impact-Aligned Social Investors

Next we analyze how the allocation of capital differs when social investors are impact-aligned. As there are multiple equilibria, in this section we focus on the one the investor-optimal equilibrium. The propositions in Section 4.3 hold across all equilibria.

We first observe that unlike in the case of values-aligned social investors, impact-aligned social investors do co-invest with commercial investors.

Lemma 2. *In the investor optimal equilibrium when social investors are impact-aligned, all firms raise at least k_i^C capital from commercial investors.*

If an impact-aligned social investor were to marginally undercut a commercial investor, they would earn a return of r^C, and create 0 additional social value. Therefore, the most efficient way to create impact is to leverage commercial capital, rather than displacing it. Firms that receive social investment raise at least k_i^C capital from commercial markets, and the remaining capital from social investors. In fact, as we demonstrate in the proof of Lemma 2, in equilibrium firms supported by social investors may raise more than k_i^C commercial capital. In such settings impact-aligned social investors subsidize the entry of commercial investors to increase the scale of firms with high marginal social value.

As in the case with binary projects, because impact-aligned social investors care about total social value creation rather than the social value of the firm they support, they do not compete with one another or with commercial investors. Rather than being determined by competitive forces, equilibrium prices of capital are determined by a no-rents condition.

Lemma 3. *Across all equilibria when social investors are impact-aligned, all entrepreneurs earn a payoff of q_i^C.*

Impact-aligned social investors demand a return r_i^S that solves $\pi_i\left(\bar{k}_i\right) - r^C\bar{k}_i^C - r_i\left(\bar{k}_i - \bar{k}_i^C\right) = q_i^C$, where $\bar{k}_i^C$ is the amount of commercial capital raised by firm i in equilibrium. If social investors demanded a higher return, firm owners would prefer to invest at their commercial scale k_i^C and to rely

at cost of capital r^C.

exclusively on commercial capital. And because impact-aligned social investors recognize that by undercutting one another they are not contributing to total social value creation, required returns are set so as to make entrepreneurs indifferent between accepting social capital versus relying exclusively on commercial financing.

We now turn to characterizing the use of minimum-scale contingencies.

Lemma 4. *Impact-aligned social investors utilize minimum-scale contingencies in equilibrium.*

Because impact-aligned social investors set prices so as to leave entrepreneurs with their commercial payoff, the firms they support are always faced with a marginal cost of capital that is above their marginal return on investment at $\bar{k}_i$. If they were free to choose their own scale, they would accept the subsidized social capital in lieu of commercial capital, and still choose a smaller scale than social investors desired. Therefore, unlike in the case with values-aligned investors, impact-aligned social investors always utilize scale-contingent contracts.

Finally, we turn to equilibrium capital allocation.

Lemma 5. *In the investor optimal equilibrium when social investors are impact-aligned, for any two firms i and j that receive capital from social investors, we have*

$$\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) = \pi_j'\left(\bar{k}_j\right) + \theta w_j'\left(\bar{k}_j\right).$$

Impact-aligned social investors allocate their capital so as to equalize the marginal profits plus the marginal social value of all firms that receive a subsidy.

Security Design

We close this section with a discussion of security design. In equilibrium, impact-aligned social investors co-invest with commercial investors but utilize different terms. Impact-aligned social investors offer cheaper financing, and include a scale contingency requiring the firm to operate at a certain scale if it is to accept their capital. This contingent financing scheme resembles the idea of green bonds, in that they are a means to offer subsidized capital to firms, earmarked for projects with high social or environmental value, without displacing commercial investors. The primary distinction is that social investors in our model support levels of output that would not have been financed by commercial investors. Therefore, a practical implementation of this market would verify that each project has negative net present value at its commercial cost of capital in addition to verifying that the project has high social or environmental value.

4.3 Results

In this section we demonstrate that natural analogues of Propositions 1 and 3 extend to this setting. Namely, we demonstrate that in equilibrium values-aligned investors leave both money and impact on the table, and that increasing a firm's profitability may also increase its enterprise impact.

First we consider our analogue to Proposition 1.

Proposition 4. *In equilibrium there may exist a deviation for a values-aligned social investor that would result in higher financial return and increase total welfare.*

When there is an intensive margin of scale, values-aligned investors still crowd out commercial capital to finance firms with high social value. Even if doing so increases the scale at which the firm operates and thus increases the social value it creates, competition among social investors means that this involves a financial concession. Again, following the same logic as Proposition 1, investors can increase their return and impact by instead supplying their capital to the firm that can most efficiently generate impact on the margin for a given financial concession.

Proposition 4 is stated as a possibility result, rather than generically as in Section 3, because when firms have an intensive margin of scale there is no natural analogue of the sufficient density assumption about the distribution of firms that was employed in Proposition 1. Therefore, even though values-aligned investors make a financial concession that does not contribute to social value, we cannot guarantee that there exists a firm to which they can unilaterally deviate to both increase their financial return and total social welfare.

Next we consider an analogue to Proposition 3. We extend the definition of enterprise impact to account for the possibility that firm i attract capital from more than one type of investor. We define the enterprise impact of firm i to be $e_i(k) \equiv w_i(k) - \nu_i k$, where ν_i is now the *average* social value of capital utilized by entrepreneur i. We have the following result.

Proposition 5. *Suppose firm i attracts financing from an impact-aligned social investor in equilibrium. Increasing its profitability while holding fixed its social value $w_i(\cdot)$ increases its enterprise impact e_i and total social value created in equilibrium.*

The logic proposition is exactly parallel to that of Proposition 5. Take any entrepreneur supported by impact-aligned social investors in equilibrium. As it becomes more profitable, commercial investors will finance a larger fraction of its output, which frees impact-aligned social capital to invest elsewhere and increase total social value.

5 Extension: Heterogeneous Investor Altruism θ

In this section we consider an extension of the model in which we allow the altruism parameter θ to vary across investors. Our aim is to explore the model's implications with regards to assortative matching. A classic exercise in the assignment matching literature is to identify conditions under which agents exhibit positive assortative matching (e.g. Roy, 1951, Becker, 1973, Sattinger, 1979, Costinot and Vogel, 2010, Gola, 2020) -– i.e. when do agents with higher "types" match with one another? We demonstrate when social investors in our model are values-aligned, investors with higher altruism match with entrepreneurs with higher social value for familiar reasons. In contrast, when investors are impact-aligned, they exhibit a variant of *negative* assortative matching. This latter result arises from the fact that impact-aligned social investors have interdependent utility; their utility depends not only on the terms of their own match but also on the matches of other investors. That impact-aligned investors exhibit negative assortative matching can be viewed as a direct extension of the logic that that they do not want to displace commercial investors with no concern for social value. Not only do impact-aligned social investors not want to displace commercial investors, but impact-aligned social investors with high levels of altruism also do not want to displace those with lower level of altruism.

5.1 Model

The model is the same as in Section 2 with the exception that for the set of social investors we now index their altruism parameter θ_i by i, and let it vary across investors. Specifically we assume that there is a finite set $\Theta \equiv \left\{\theta^1, \ldots, \theta^n\right\}$ of potential levels of altruism, with $\theta^j < \theta^k$ for $1 \leq j < k \leq n$. We make no assumption about the distribution of θ_i. We now let S^l be the set of social investors with altruism parameter θ^l. We maintain all other assumptions of the model in Section 2.

5.2 Values-Aligned Social Investors

We now characterize the equilibrium of the model where all social investors are values-aligned and demonstrate that social investors and entrepreneurs exhibit positive assortative matching on θ and w.

Prices of capital offered to any two entrepreneurs i and j who are both supported by a social investor with type θ^l satisfy $r_i + \theta^l w_i = r_j + \theta^l w_j$.[25] And for an entrepreneur i supported by a social investor

[25]The preceding equality holds so long as prices are finite. In equilibrium a social investor may provide funding in exchange for zero share of the proceeds ($r = 0$) if the project has sufficiently high social impact (akin to philanthropy). In such a case, the above equality need not hold.

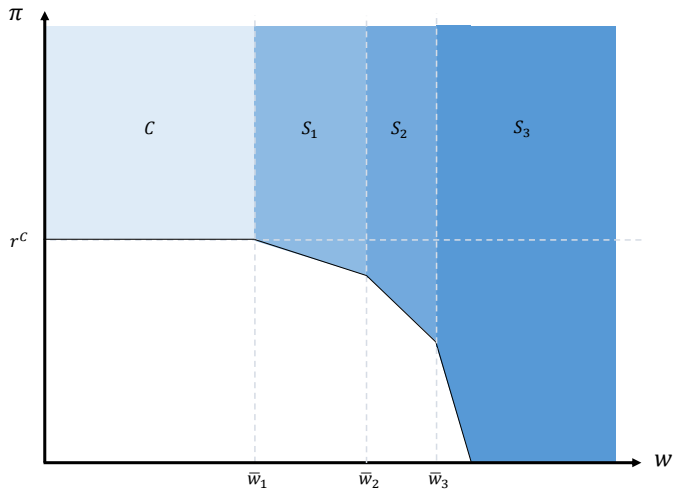

Figure 5: Equilibrium sorting with values-aligned social investors

of type θ^l and an entrepreneur k who is not, prices of capital must satisfy $r_i + \theta^l w_i \geq r_k + \theta^l w_k$.

With the above pricing equations we can characterize the set of entrepreneurs financed by each type of investor in equilibrium. The equilibrium investment allocation is depicted in Figure 5.

Relative to Section 3.1 the principle novelty is that we can now establish assortative matching in equilibrium. Namely, investors partition the set of entrepreneurs who receive financing such that investors with higher θ_i match with entrepreneurs who have higher w_i. This stems from the fact that the utility of investor i is super-modular in θ_i and w_i, and hence social investors with higher altruism have a higher willingness to pay for projects with high social value. This positive assortative matching echoes many results in the assignment matching literature cited above. As we will see in the following section, this result breaks down, and partially reverses when social investors are impact-aligned.

5.3 Impact-Aligned Social Investors

When social investors are impact-aligned there is a multiplicity of equilibria; Figure 6 depicts the investment allocation in the investor- and welfare-optimal equilibrium. Appendix Section A.5 offers a formal characterization of this equilibrium.

Relative to when social investors are values-aligned, the equilibrium allocation features two important differences. First, as in Section 3.2, so long as social capital is sufficiently scarce, impact-aligned social investors exclusively support firms that could not attract commercial financing. Second, and novel to this section, positive assortative matching breaks down, *even among the set of firms supported by social investors.* In fact, holding fixed a level of profits π, social investors exhibit negative

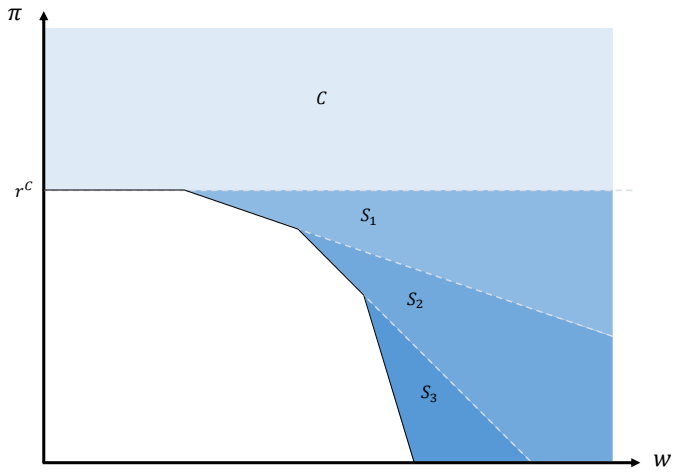

Figure 6: Equilibrium sorting with impact-aligned social investors

assortative matching; the higher is the social investor's altruism parameter θ_i, the lower is the social value w_i of the firm they support. This negative assortative matching holds despite the fact that the utility of impact-aligned social investors is still super-modular in their altruism parameter θ_i and the social value w_i of the firm they support.[26]

In equilibrium, in order for an impact-aligned social investor i not to deviate and support a firm that could have attracted commercial investment it must be that

$$\pi_i + \theta_i w_i \geq r^C.$$

This incentive compatibility condition is easier to satisfy for social investors with higher altruism parameters. Therefore in the welfare optimal equilibrium, it is the impact-aligned social investors who care the least about social welfare that match to the most impactful entrepreneurs for a given level of profitability, as these are the entrepreneurs who are most able to entice social investors away from commercial markets. In contrast, impact-aligned social investors with higher altruism parameters are willing to forgo commercial returns to support entrepreneurs with lower contribution to social welfare for a given profit level. And because impact-aligned social investors derive utility from the social value created by all firms supported in equilibrium, social investors with high altruism parameters do not compete with social investors with low altruism parameters, as they recognize that doing so would not expand social value. Therefore, that positive assortative matching breaks down, and partially reverses

[26] All equilibria with the same investment frontier depicted in Figure 6 are investor- and welfare-optimal. Therefore, formally, there exists an investor- and welfare-optimal equilibrium such that holding the level of entrepreneur profit fixed, social investors engage in negative assortative matching. But there may be other equilibria with equivalent allocations that do not feature negative assortative matching.

when social investors are impact-aligned arises from the fact that impact-aligned social investors have interdependent utilities in the sense that their utility depends not only on the firm they match to and their financial return but also on the matching of other investors and entrepreneurs. This result can be understood as an extension of the core logic that impact-aligned social investors do not want to displace investors with less concern for social welfare, whether they be commercial investors or other impact-aligned social investors with a lower degree of altruism.

6 Empirical Evidence and Practical Implications

The fundamental message of this paper is that investors who care about impact should not pay a premium to make investments that could have been financed by any investor less interested in social impact. In our model, values-aligned investment strategies lead to exactly such a premium. This creates a deviation by which a social investor can increase both financial return and aggregate social value. The extent to which these insights matter and are applicable to the real world depend on three factors. First, do social investors adopt values-aligned investment strategies? Second, do social investors actually have preferences for social value *creation* rather than social value *association*? And third, are these investors indeed paying a "greenium" for their investments in high social value companies that could have been financed with commercial capital? Further, what can social investors actually do in practice to increase their impact? In this section we review the literature and present new empirical analysis that sheds light on these questions.

6.1 Strategies of Social Investors: Evidence from US Equity Mutual Funds

Are socially minded investors adopting values-aligned or impact-aligned investment strategies? Figure 3 provides guidance for answering this question. Values-aligned investors should exhibit a preference for investments associated with higher social value, as proxied by companies having higher ESG score, for example. In contrast, impact-aligned investors may also favor these companies, but will place special emphasis on the socially valuable firms that are not sufficiently profitable to easily attract commercial capital.

To operationalize this insight we study the portfolios of U.S. equity mutual funds, comparing holdings of funds that describe themselves as having sustainability goals or orientations to the broader universe of mutual funds.[27] We begin with all US mutual funds with Morningstar fund ratings as of

[27]Mutual funds have a fiduciary duty to invest only for the *financial* benefit of their clients. It is thus unlikely a mutual fund will explicitly state it has an objective of social value creation. Instead, these funds often argue that investment strategies that generate social impact will also generate attractive financial returns.

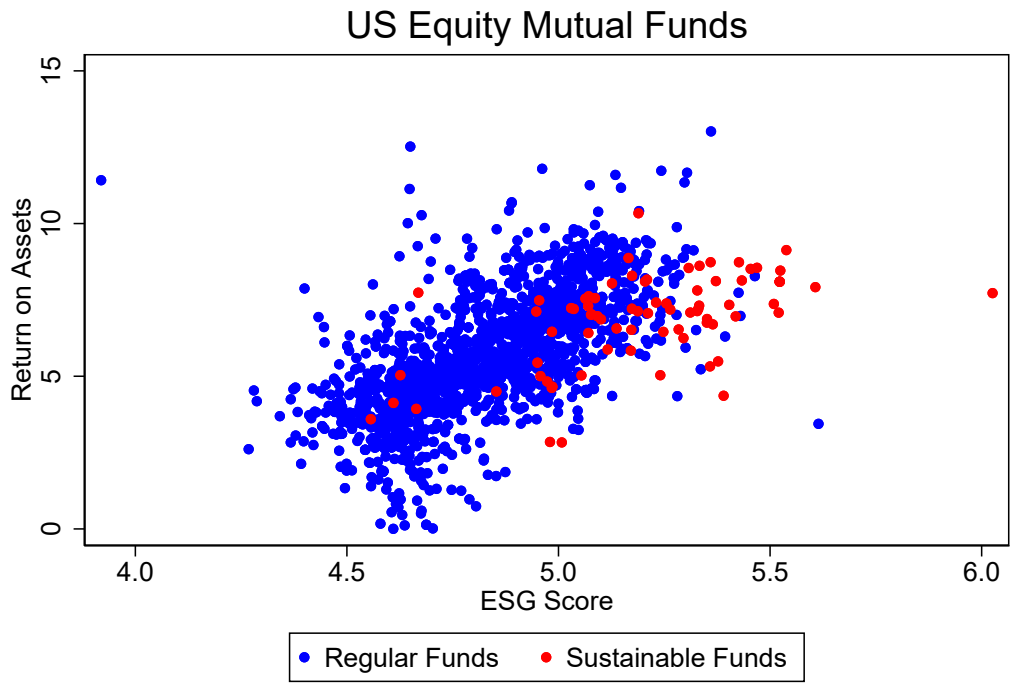

Figure 7: Portfolio Characteristics of Equity Mutual Funds

2019. To maximize data concordance, we further limit the sample to funds that Morningstar classifies as primarily investing in U.S. Equities. To study the portfolio allocation of funds, we also require funds in our sample to match to 2019Q4 security-level holdings data in the CRSP Mutual Fund Holdings Database. We combine this fund-level data with security-level data on sustainability scores from MSCI ESG Research, financials from CapitalIQ, and consensus analyst earnings forecasts from I/B/E/S. As a further restriction on the sample of mutual funds we limit to funds for which we can match at least 75 percent of their holdings by volume to our company-level financial and sustainability data.

Finally, we designate a fund as "sustainable" if it is one of the 303 mutual funds identified by Hale (2020) as an explicitly sustainable mutual fund active in 2019. This classification is designed to focus on mutual funds that go a step further than merely mentioning social considerations in their prospectuses (of which Hale (2020) counts 564 in 2019, up from only 81 in 2018). Funds designated as sustainable must make sustainability an "intentional focus" and central part of the investment strategy. Our final sample consists of 1,870 mutual funds, of which 82 are classified as sustainable.

Table 1 provides summary statistics of the mutual fund data used in the analysis. The (value-weighted) portfolio holdings of sustainable mutual funds are distinct in several dimensions. Unsurprisingly, sustainable funds are younger, smaller, and hold stocks with higher ESG ratings. More

Table 1: Summary Statistics of Mutual Fund Sample

	(1)	(2)	(3)
		Means	
Variable	Full Sample	Sustainable Funds	Difference
Fund Size (millions)	3,682	1,731	-2,041
	(16,256)	(4,092)	(1,883)
Number of Securities Held	197	193	-4
	(269)	(192)	(31)
Fund Age	16.5	10.4	-6.4***
	(13.6)	(10.6)	(1.6)
Morningstar Fund Rating	3.17	3.38	0.21
	(1.03)	(1.00)	(0.14)
Average Holding ESG Score	4.86	5.20	0.36***
	(0.22)	(0.25)	(0.02)
Environmental Score	5.49	6.11	0.65***
	(0.87)	(0.72)	(0.10)
Social Score	4.52	4.90	0.39***
	(0.28)	(0.30)	(0.03)
Governance Score	5.89	5.96	0.07***
	(0.19)	(0.16)	(0.02)
Return on Assets	5.79	6.85	1.11***
	(2.06)	(1.50)	(0.24)
Earnings Growth Forecast	11.40	10.51	-0.93**
	(3.68)	(2.46)	(0.43)
Observations	1,758	78	1,758

Notes: This table reports summary statistics for the sample of mutual funds used in the empirical analysis in Section 6. The sample and data are constructed based on end of year 2019 data. A mutual fund is in the sample if it has a Morningstar rating, is classified by Morningstar as investing primarily in U.S. Equities, can be matched to the CRSP Mutual Fund Holdings Database, and for which we can match at least 75 percent of holdings by volume to sustainability and financial data from MSCI ESG and Capital IQ, respectively. A fund is classified as "Sustainable" if it is on the list of sustainable mutual funds in Hale (2020). The table reports group means and differences, as well as standard deviations in parenthesis.

Table 2: Determinants of Sustainable Fund Ownership of Securities

Dependent variable =	Sustainable Fund Relative Portfolio Weight				
	(1)	(2)	(3)	(4)	(5)
ESG Score	0.708***			0.710***	0.739***
	(0.089)			(0.088)	(0.097)
Environmental Score		0.158***			
		(0.023)			
Social Score		0.195***			
		(0.037)			
Governance Score		0.090*			
		(0.053)			
Profitability			0.011	-0.000	0.005
			(0.010)	(0.010)	(0.009)
Growth Expectations			0.005	0.005	0.008
			(0.005)	(0.005)	(0.005)
Industry FE	No	No	No	No	Yes
Number of Companies	1,342	1,342	1,342	1,342	1,342
R^2	0.072	0.046	0.003	0.074	0.132

Notes: This table reports cross-sectional regressions describing the relationship between attributes of US equity issuers and the extent to which their stock is held by a sample of "Sustainable" mutual funds, as identified by Hale (2020). The dependent variable is the ratio of a stock's weight across the aggregate portfolio of "Sustainable" mutual funds relative to the stock's weight in the overall mutual fund sample. *ESG Score*, and the three component subscores, are company-level metrics compiled by MSCI ESG Research as of 2019. *Profitability* is the company's return on assets in 2019 and *Growth Expectations* is the I/B/E/S median consensus forecast for long term earnings growth. Both variables are measured as percentages. Column 5 controls for industry fixed effects. Robust standard errors are reported in parenthesis.

interestingly, sustainable mutual funds hold companies that have a higher return on assets – our proxy for profitability – and lower analyst growth expectations.

Figure 7 plots the portfolio weighted average ESG score and return on assets of mutual funds in our sample. This is a close empirical analogue to our equilibrium characterization in Figure 3; profits are on the y-axis and social value is on the x-axis. For the most part, sustainable mutual funds appear to act as if they are values-aligned. At least on average, sustainable mutual funds do not invest in companies that are less profitable and thus less able to raise commercial capital from commercial investors.

This result can also be seen at the company rather than mutual fund level. Table 2 reports regression results exploring which company characteristics are associated with overweighting in sustainable fund portfolios. These cross-sectional regressions are estimated on the sample of publicly traded companies for which we have sufficient financial and ESG score data and are able to match to our data on mutual fund holdings. The dependent variable is the weight of the stock in the aggregate portfolio of sustainable mutual funds relative to the stock's portfolio weight in the broad sample of funds. A value of one means sustainable funds hold a stock in the same proportion as the broader sample of mutual funds, and a value of two indicates sustainable funds hold a stock in double the weight of the average mutual fund. Table 2 shows that ESG scores alone, and not measures of profitability and growth, are the only economically and statistically significant predictors of sustainable mutual fund holdings. Thus, there is no detectable evidence that sustainable mutual funds take into consideration the ability of the companies they invest in to attract commercial capital. As sustainable mutual funds behave as if they are values-aligned, our theoretical results may offer guidance as to how they can increase both their financial returns and impact.

6.2 The Preferences of Social Investors

The previous analysis showed US Equity mutual funds describing themselves as investing with sustainability considerations appear to adopt values-aligned investment strategies. Does this reflect the underlying preferences of the investors in these funds? Or, does it highlight an opportunity to better align investment strategies with the underlying objectives of investors who seek to create an impact in the world, rather than merely being associated with it? In this section we provide new narrative evidence suggesting that investors do value impact, and we survey the academic literature investigating the social preferences of investors.

A first indication that social investors value impact comes from the marketing statements of ESG and sustainable investment funds, which routinely allude to the positive impact of their investment strategies. For a few examples of many, Nuveen, an investment management firm with one trillion

dollars in assets under management, asserts that ESG investing is the approach "that is most likely to produce optimal financial and societal outcomes."[28] Candriam, a socially responsible investment fund with €130 billion under management, asserts on its webpage that they "invest in the future, channeling capital for the common good."[29] Calvert, another provider of ESG mutual funds with $23 billion in assets under management, goes one step further; their website allows investors to calculate the impact of their investment in a Calvert mutual fund across a variety of outcomes.[30] These examples indicate that ESG investment funds aim to attract investors with a direct preference for the creation of social value.

Approaching the question from a different angle, several academic papers examine the motivation of investors using laboratory experiments. Drawing on a population of real-world investors, Heeb et al. (2021) report on an experiment in which subjects were asked their willingness to pay for an asset, and the authors varied both the asset's financial return and environmental impact. The authors find that willingness to pay is higher for assets with a positive impact, but that willingness to pay is insensitive to the level of impact. While the authors did not vary whether investment in the asset creates an impact or is merely an association with impact, this evidence still suggests that investors do not have fully impact-aligned preferences. Bonnefon et al. (2019) report on a similar exercise with a convenience sample of respondents drawn from Amazon's Mechanical Turk. The authors elicit respondents' willingness to pay for a synthetic asset that offers both a financial return and a donation to charity. Counter to the findings of Heeb et al. (2021), the authors find that willingness to pay does scale with the impact of the asset. However they also find that varying whether respondents are pivotal in the impact, or whether they are merely paying for association with the impact does not influence their willingness to pay, suggesting that their sample was primarily motivated by values association.

Outside of the investment context, a large literature investigates whether donors are motivated by warm glow (what we refer to as values-alignment) or pure altruism (what we refer to as impact-alignment). One of the most popular tests of whether donors are pure altruists relies on the extent to which a subject's donations are crowded out by the donations of others to the same cause. If a subject is only motivated by her own consumption and total social impact, then if a social planner (or laboratory experimenter) were to tax the subject by $1 and donate it to charity, that should crowd out her subsequent donation to the same charity by exactly $1. In contrast, if she values being associated with the output of the charity, then taxing her $1 and reallocating it to the charity should reduce

[28] See: https://www.tiaa.org/public/pdf/how_nuveen_uses_responsible_investing_across_asset_classes.pdf Last Accessed: November 27, 2020

[29] See: https://www.candriam.com/en/private/about-us/ Last Accessed: November 27, 2020

[30] In fact, Calvert's calculation of the impact of an investment conflates measures of social value of portfolio companies with the impact of an investor in these companies. For example, it reports that a $10,000 investment in Calvert US Large-Cap Core Responsible Index Fund results in an annual reduction in emissions equivalent to burning 147 gallons of gasoline. This figure is based on the difference between the value-weighted emissions of constituents in the Calvert fund and the Russell 1000 Index. See: https://www.calvert.com/what-is-your-impact.php Last Accessed: November 27, 2020

her donation by less than $1. A number of papers find experimental evidence of imperfect crowd out (see e.g. Andreoni (1993), Bolton and Katok (1998), Chan et al. (2002), and Gronberg et al. (2012)) indicating that warm glow is a motive in charitable giving. Ottoni-Wilhelm et al. (2017) conduct an experiment that utilizes this same crowding out test, but does so at varying baseline levels of donations to a particular charity. The authors demonstrate that this allows them to quantify the relative importance of warm glow and pure altruism as motives for giving, and they conclude that pure altruism is the more important factor among their sample.

While the evidence above paints a mixed picture about the nature of altruism, our assessment is that it suggests that impact-alignment (or pure-altruism) is an important factor in the motivations of social investors.

6.3 Is There a Greenium for High Social Value Companies?

The final ingredient necessary for our main result is that investors bid up the prices of socially valuable companies, or equivalently lower their cost of capital, relative to the valuations and costs of capital that these companies would have achieved in the absence of social investors. Detecting such effects in the data is complicated by the fact that, regardless of social motivations, there is substantial debate about the degree to which the social value of companies is associated with fundamental risks, and if these risks are correctly priced (e.g., Andersson et al. (2016).)

Unsurprisingly, the evidence is mixed. Studies measuring ex-post performance of companies ranking highly in sustainability or social responsibility measures often find they outperform other stocks on a risk-adjusted basis, i,e. Eccles et al. (2014). In general, such out-performance could come from a higher cost of capital, changes in fundamentals, or a decrease in the required return of investors going forward. Other studies find it is *low* social value companies that outperform (Hong and Kacperczyk (2009)). Using ex-ante cost of capital measures, El Ghoul et al. (2011) find socially responsible companies face a lower cost of equity capital than other companies by 56 basis points. Studying green bonds, Baker et al. (2018) finds a small "greenium" in municipal bonds, while Larcker and Watts (2020) and Flammer (2021) find no premium for green bonds in municipal and corporate bond markets, respectively. Flammer (2021) does find that the issuance of green bonds is associated with a positive equity price reaction and an increase in equity ownership by "green" investors. Hartzmark and Sussman (2019) show that the introduction of Morningstar mutual fund sustainability ratings lead to significant flows into funds with high sustainability ratings and out of funds with low sustainability ratings, consistent with investor preference for this attribute, but did not find significant pricing effects.

While there is no clear consensus that socially valuable companies face a lower cost of capital,

measurement issues are a significant obstacle to resolving this debate. Existing studies have not isolated potential pricing effects arising from investors' non-fundamental taste for these assets from other confounding determinants of valuation and cost of capital.

6.4 Practical Considerations

Our theoretical results suggest that social investors could have more impact and higher financial returns if they were to reallocate their investments away from projects that are profitable at commercial rates and towards projects that require subsidy to be viable. How would this work in practice? In private markets this would principally require that impact investors not undercut socially-neutral investors when searching for companies to finance.[31] Our results do not imply that impact investors must deploy all of their capital in deals that have not attracted the interest of socially-neutral investors, but to the extent that an impact investor is not able to deploy all of their capital in this way, they have effectively exhausted their opportunities for impact (or at least those that justify the associated financial concessions).

In public markets there are, by definition, no opportunities to finance firms that have not attracted socially-neutral investment. Nevertheless, our analysis in Section 4 demonstrates that investors can have an impact by subsidizing impactful projects that would not be profitable within firms that have attracted some socially-neutral capital. As discussed in Section 4 this could be achieved through a variant of green bonds that offer project-specific financing and not only verify that the project is socially or environmentally valuable but also verify that it would not have been profitable without subsidy.

7 Conclusion

This paper provides a new framework to understand how values-based investing generates social impact. We analyze a model in which investors influence social outcomes through their asset allocation, and show that equilibrium asset allocation differs in important ways depending on precisely how social investors think about social value creation.

Investors following values-aligned investment strategies, which closely resemble the construction of conventional ESG and emissions reduction portfolios, have limited impact because they displace

[31] In our model, impact investors also do not undercut one another, however we conjecture that in a dynamic variant of the model, displacing an impact investor would still contribute to higher social value creation than displacing a commercial investor.

commercial investors who do not care about social value creation but would have supported some socially valuable companies anyway. Further, because values-aligned investors place intrinsic value on owning socially valuable firms, they compete with one another and push up the price of firms that could have been financed by commercial investors. From the perspective of generating impact, we show that this financial concession is inefficient. We identify an alternative investment approach, which only makes financial concessions to subsidize firms that would not be viable at the commercial cost of capital, and which generates more impact and also higher financial returns.

We demonstrate two further implications of the idea of that impact can arise from not displacing investors who care less about social value. First, we show that when there is heterogeneity in the altruism of impact-aligned investors, equilibrium involves negative assortative matching–high altruism investors finance lower social value projects so as to avoid displacing lower altruism investors from investing in high social value projects. Second, from a firm's perspective, we show that making a firm more profitable can also make it more impactful. Impact-aligned investors, who care about their financial return and their contribution to social value, seek not to invest in firms that could have attracted commercial capital. The more profitable a firm, the less likely it is to utilize the capital of impact-aligned investors. So taking a firm that would have been financed by an impact-aligned investor, and making it more profitable, allows it to utilize less socially valuable capital and frees up the impact-aligned capital to support a new firm.

When firms have an intensive margin of scale, impact-aligned investors' desire not to displace commercial investors also gives rise to "blended finance." Social investors can maximize their impact by co-investing with commercial investors at different terms, so as to leverage commercial investment rather than displacing it. This resembles existing "green bonds," but with an important difference. In practice green bonds must only show that the use of proceeds goes towards funding investment with desirable social attributes. Our framework highlights that these securities should be further restricted to only fund investments that are also not viable at the commercial cost of capital.

The practical importance of our findings hinges on three factors–the degree to which social investors have preference for impact creation, whether social investors follow values-aligned strategies, and whether social investors contribute to an overvaluation of companies with high social value whose projects would otherwise be profitable at the commercial cost of capital. We show empirically that a large class of social investors (ESG equity mutual funds) seem to exhibit values-aligned investment strategies in that they favor companies associated with high social value, but do not avoid investment in the most commercially viable of these firms. Surveying the literature, we find inconclusive and mixed evidence, however, on whether this reflects the true preferences of investors, and whether this behavior inflates the valuation of high social value companies.

Our analysis raises several unanswered questions. First, it highlights the importance of better

measurement of the behavior and valuation effects of socially minded investment strategies. Further, it raises the question of how investors should measure their own impact, given that the impact of an investor depends not only on the social value of the firms they support, but also on whether those firms could have received financing from value-neutral investors. These questions are fertile ground for future work.

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A Equilibrium Characterizations and Proofs

A.1 Characterization of the Equilibrium with Values-aligned Investors in Section 3.1

First, it is straightforward to show that in equilibrium, for any two entrepreneurs i and j who are both supported by a social investor, their costs of capital satisfy $r_i + \theta w_i = r_j + \theta w_j$, unless $r_i = 0$ or $r_j = 0$. If not, there is a pair of social investors i and j for whom $r_i + \theta w_i > r_j + \theta w_j$, in which case j would prefer to deviate and offer entrepreneur i a contract with cost of capital $r_i - \varepsilon$ for sufficiently small ε. Similarly, in equilibrium for an entrepreneur i supported by a social investor and an entrepreneur k who is not, prices of capital must satisfy $r_i + \theta w_i \geq r_k + \theta w_k$. Utilizing these conditions we have the following lemma.

Lemma 6. *Projects financed by social investors have higher social value than projects financed by commercial investors. Formally, if entrepreneur i is financed by a social investor and entrepreneur j is financed by a commercial investor, $w_i \geq w_j$.*

Lemma 6 implies that there is some $\bar{w}$ such that social investors only support entrepreneurs with social impact greater than $\bar{w}$ and commercial investors only support entrepreneurs with social value less than $\bar{w}$. Commercial investors support entrepreneurs with profit higher than r^C and with social impact lower than $\bar{w}$. Entrepreneurs with profit lower than r^C cannot generate sufficient profits to attract commercial support, and entrepreneurs with social impact higher than $\bar{w}$ are priced by social investors and therefore yield returns too low to attract commercial support.

Lemma 7. *For entrepreneurs with $w_i \geq \bar{w}$, social investors support those with $\min\{\pi_i, r^C\} + \theta w_i \geq r^C + \theta\bar{w}$ and the price they pay is such that either $r_i + \theta w_i = r^C + \theta\bar{w}$ if such an $r_i \geq 0$ exists and $r_i = 0$ else.*

Because commercial investors support all entrepreneurs with profit higher than r^C and with social value lower than $\bar{w}$, $r^C + \theta\bar{w}$ is the social investor's effective outside option utility. Prices in equilibrium are set such that investors achieve this outside option utility.

To support this equilibrium allocation, each entrepreneur i who receives financing from a social investor at cost r_i must receive at least two such offers. If not, the social investor who supports entrepreneur i would be incentivized to deviate and offer financing at cost r^C instead. To see that a social investor has no incentive to deviate from the offer of r_i when an entrepreneur has at least two such offers, note that by construction

$$r^C + \bar{w} \leq r_i + w_i$$

A direct implication is that

$$\frac{1}{2}\left(r^C + w_i\right) < r_i + w_i$$

which is the incentive compatibility constraint for the social investor of entrepreneur i when i receives two offers. Therefore, in equilibrium, while every social investor's offer is accepted by one entrepreneur, each entrepreneur with social financing receives at least two offers (and accepts one).

Social investors receive financial return $r^C - \theta\left(w_i - \bar{w}\right)$. Thus, they are willing to pay (in terms of reduced financial return) for projects that generate high social value. Commercial investors do not find it attractive to invest in companies with $\pi_i > r^C$ and $w_i > \bar{w}$ precisely because social investors are willing to invest in these companies at higher valuations.

Lemma 7 allows for the graphical characterization of the equilibrium, depicted in Figure 1. That $\bar{w}$ is uniquely pinned down is implied by the resource constraints, i.e. that social investors have mass S.

A.2 Characterization of the Equilibrium with impact-aligned Investors in Section 3.2

As impact-aligned social investors care about total social welfare rather than the social value of the entrepreneur they support, their expectations about which entrepreneurs will be financed in the commercial market will determine whom they finance. In equilibrium, social investors and entrepreneurs expect that those entrepreneurs with profits above r^C can receive commercial financing and earn $\pi_i - r^C$. We have the following lemma regarding the prices of firms supported by impact-aligned social entrepreneurs.

Lemma 8. *For any entrepreneur i supported by a social investor, $r_i = r^C$ if $\pi_i \geq r^C$ and $r_i = \pi_i$ else.*

Using the equilibrium prices from Lemma 8, we can characterize the set of entrepreneurs supported by social investors.

Define $s_w \equiv \left\{i \in E : \pi_i \leq r^C, \pi_i + \theta w_i \geq r^C + \theta w\right\}$. If there exists a $\tilde{w} > 0$ such that $|s_{\tilde{w}}| = |S|$, then in equilibrium the social investors support entrepreneurs in $s_{\tilde{w}}$ and have a strict preference for these firms over supporting any firm with $\pi_i > r^C$. Prices for firms in $s_{\tilde{w}}$ are set such that $p_i = \pi_i$.

To see that this behavior can be supported in equilibrium, there are three deviations we must consider for impact-aligned social investors – supporting an entrepreneur j with profit $\pi_j > r^C$, who will get commercial financing, supporting an entrepreneur $k \in s_{\tilde{w}}$ who already receives support from another social investor, or supporting an entrepreneur l who does not receive any investment in equilibrium.

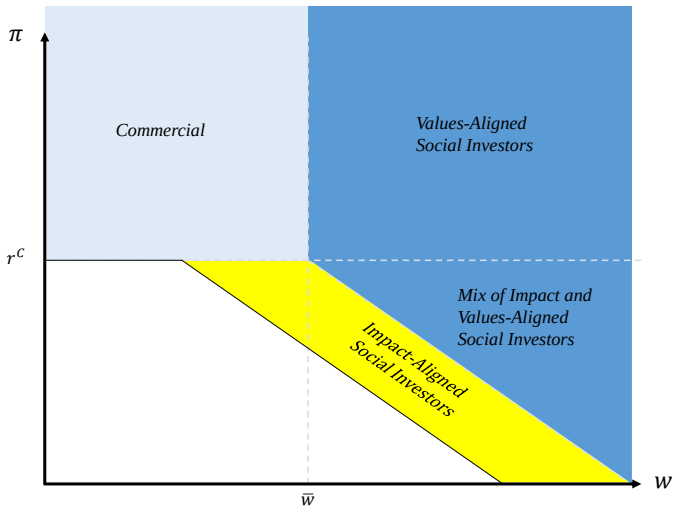

Figure 8: General Equilibrium With Both Types of Social Investors

In the first case, the social investor cannot earn more than r^C profit, as this is what the commercial investor earns. Moreover, she recognizes that by undercutting a commercial investor, she is not contributing to total social value, and so by definition of S_w this is not a profitable deviation.

In the second case entrepreneur k produces $\pi_k \leq r^C$ profit and the social investor recognizes that by undercutting another social investor she is not contributing to the total social welfare created because entrepreneur k will be financed independently of her actions, and the displaced social investor cannot reallocate her capital. Therefore by definition of S_w these are not profitable deviations. In the final case, the most attractive entrepreneur to support is the one with profit $\pi_l = r^C$ and social impact $w_l = \tilde{w}$, but once again by definition of S_w this is not a profitable deviation.

If there is no $\tilde{w} > 0$ such that $|s_{\tilde{w}}| = |S|$, the in equilibrium impact-aligned social investors support all firms in s_0, and allocate the remainder of their capital either to firms with profits $\pi_i > r^C$ or to the outside option asset.

A.3 Characterization of the Equilibrium with Both Types of Social Investors in Section 3.3

Figure 8 depicts the equilibrium structure. To characterize this structure we use the following three lemmas.

Analogous to Section 3.1 there exists a $\bar{w}$ such that

Lemma 9. *Commercial investors support the entrepreneurs with $\pi_i \geq r^C$ and with $w_i \leq \bar{w}$.*

Analogous to Lemma 7 we have

Lemma 10. *Values-aligned investors support entrepreneurs with* $\min\left\{\pi_i, r^C\right\} + \theta w_i \geq r^C + \theta\bar{w}$.

Finally, analogous to the equilibrium characterized in Section A.2, there exists a $\tilde{w} \geq 0$ such that

Lemma 11. *Impact-aligned social investors support entrepreneurs with profits* $\pi_i < r^C$ *and with* $\pi_i + \theta w_i > r^C + \tilde{w}$. *If* $\tilde{w} > 0$ *then the former inequalities completely characterize the set of entrepreneurs supported by impact-aligned social investors. If* $\tilde{w} = 0$ *then some impact-aligned social investors utilize the outside option asset.*

A few features of the equilibrium are of note. First, both impact-aligned and values-aligned investors may support firms in the region below the horizontal line $\pi = r^C$ and to the right of the diagonal line $\pi_i + \theta w_i \geq r^C + \theta\bar{w}$. This is because in equilibrium, impact-aligned investors know they are pivotal for the firms they support in this region, since no values-aligned social investor would support them if the impact-aligned investor were to deviate.

Second, only values-aligned social investors support firms in the region above $\pi = r^C$ and to the right of $w = \bar{w}$ because these are firms that could attract commercial support if only a values-aligned investor did not support them, and so by the same logic as in Section 3.2 these firms cannot attract the support of impact-aligned social investors.

Next we argue that the equilibrium depicted in Figure 3 is welfare-optimal. This is the equilibrium in which only values-aligned social investors support firms in the region below the horizontal line $\pi = r^C$ and to the right of the diagonal line $\pi_i + \theta w_i \geq r^C + \theta\bar{w}$. It is straightforward to show that the set of firms supported in this equilibrium contains the set of firms supported in all other equilibria. Because social investors never support firms with $w_i < 0$, as these are dominated by the value-neutral outside option asset, it follows that this is the equilibrium with highest welfare.

Next we argue that the equilibrium depicted in Figure 3 is investor-optimal. This is the equilibrium in which only values-aligned social investors support firms in the region below the horizontal line $\pi = r^C$ and to the right of the diagonal line $\pi_i + \theta w_i \geq r^C + \theta\bar{w}$. It is straightforward to show that this is the equilibrium where $\bar{w}$ is the highest. To see that the sum of the financial returns is highest in this equilibrium, note that only values-aligned social investors place competitive pressure on prices, and prices for all firms to the right of the diagonal line $\pi_i + \theta w_i \geq r^C + \theta\bar{w}$ satisfy $r_i + \theta w_i = r^C + \theta\bar{w}$. Because $\bar{w}$ is the highest, prices are also the highest in equilibrium. To see that social welfare is the highest in this equilibrium, note that he set of firms supported in this equilibrium contains the set of firms supported in all other equilibria. Because this equilibrium has features both the highest total social investor returns, and the highest social welfare, it is the equilibrium in which the sum of social investor welfare is maximized (and commercial investors welfare is constant across all equilibria).

A.4 Omitted Proofs From Sections 3.3 and 4

Proof of Propositions 1 and 2

First, we provide a formal definition of "sufficient density" referenced in Proposition 1. We say that the distribution of firms is ε-*dense* if for every $\pi \in [0, \hat{\pi}]$ and for every $w \in [-\hat{w}, \hat{w}]$, there is a firm i with $|\pi_i - \pi| + |w_i - w| < \varepsilon$. Proposition 1 is formally stated as follows: *Consider any values-aligned social investor i that supports a firm with $\pi_i > r^C$ and earns a return $r_i < r^C$ in equilibrium. There exists an $\varepsilon > 0$ such that If the distribution of firms is ε-dense, there exists an unfinanced firm j with profits $\pi_j > r_i$, such that if the values-aligned social investor i were to deviate and offer firm j financing at cost π_j, total social welfare would increase as would investor i's financial return.*

We sketch the proof of Proposition 1 with reference to Figure4, where a values-aligned investor in the blue region is moved to instead support an entrepreneur in the green region. For any level of financial compromise that a values-aligned social investor makes to support a firm that could have attracted commercial financing, ε-density guarantees that there exists a firm that is not supported by any investor but that could offer a higher return than the values-aligned social investor is earning, and such that if the values-aligned social investor were to reallocate her capital to the new firm total social welfare would increase. This new firm has lower w_i than the one that the values-aligned social investor supported, but the values-aligned social investor's contribution to social welfare is higher when supporting the new firm because it could not attract commercial financing.

We sketch this proof of Proposition 2 with reference to Figure 9. First we identify a set of values-aligned social investors who are earning 0 return in equilibrium. We convert these values-aligned social investors to impact-aligned social investors, noting that this strictly expands the set of firms supported by impact-aligned social investors. We make no assumption about which of the firms supported by impact-aligned social investors are supported by our "converted" investors. Instead we note that in equilibrium all impact-aligned social investors earn positive profits, as the price for each of these firms is $r_i = \pi_i$. Therefore our converted investors earn higher profits than prior to their conversion. That social welfare is higher is a straightforward consequence of the fact that strictly more firms now receive financing, and that social investors never finance firms with $w_i < 0$, as these are dominated by the value-neutral outside option asset.

Proof of Proposition 3

This is a straightforward implication of the equilibrium depicted in Figure 3. Fix any entrepreneur i who is being supported by an impact-aligned social investor. Raising his profit π_i to $\pi_j < r^C$ does

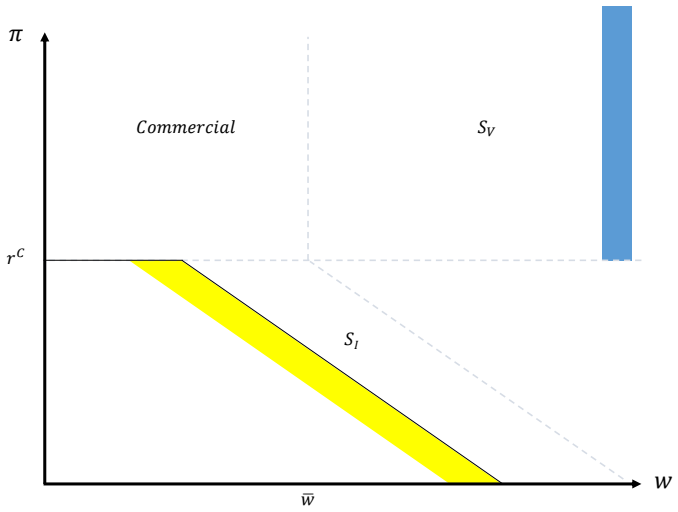

Figure 9: Converting Values-aligned to Impact-aligned Investors

not change the type of capital he attracts. But raising his profit to $\pi_j > r^C$ causes him to instead be supported by a commercial investor and his enterprise impact increases. Social welfare increases because the impact-aligned social investor can now support another entrepreneur.

Proof of Lemma 1

In any equilibrium all values-aligned social investors are indifferent between supporting any two firms with average social value exceeding $\bar{w}$. Suppose that for one such firm i, where a social investor j offers a cost of capital $r_i < r^C$ there was co-investment. That is, the firm receives some capital at $r_i < r^C$ and some capital at r^C. Then a social investor supporting another firm could deviate and offer firm i a unit of capital at $r_i' = r_i + \varepsilon < r^C$, thereby displacing one unit of commercial capital that firm i previously accepted. This social investor would have strictly higher utility than social investor j, violating the equilibrium indifference condition.

Proof of Lemmas 2, 3, 4, and 5

We directly construct the investor-optimal equilibrium. Let $\bar{k}_i$ be the equilibrium scale of firm i, representing the total social capital it is offered in equilibrium plus $\bar{k}_i^C$, which represents the amount of commercial capital firm i raises in equilibrium. Each social investor who offers firm i a contract specifies the minimum-scale contingency $\bar{k}_i$, and the required return r_i^S solving $\pi_i\left(\bar{k}_i\right) - r^C\bar{k}_i^C - r_i^S\left(\bar{k}_i - \bar{k}_i^C\right) = q_i^C$.

There is a unique allocation of capital such that

1. $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) = \pi_j'\left(\bar{k}_j\right) + \theta w_j'\left(\bar{k}_j\right)$ for all firms i and j that receive social capital
2. $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) \geq \pi_k'\left(\bar{k}_j\right) + \theta w_k'\left(\bar{k}_j\right)$ for firm i that receives social capital and firm k that does not
3. Either,
   (a) For all firms that receive social capital, $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) = r^C$, or
   (b) $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) > r^C$ and $r_i^S = 0$.

Social investors offer $\bar{k}_i - k_i^C$ capital to firm i and charge r_i^S satisfying 1-3 above.

We first verify that this is an equilibrium and second verify that it is the investor-optimal equilibrium.

To see that it is an equilibrium, note that it is incentive compatible for firms to accept all contracts that they are offered, as they earn q_i^C regardless of whether they accept all of their social investment contracts and operate at $\bar{k}_i$ or whether they accept only the commercial capital and operate at k_i^C. To see that this is incentive compatible from the investor's perspective, note that the marginal social return plus the marginal profits are equalized across all firms that receive social investor financing, and higher than all of those that do not. If a social investor were to deviate, she would gain at most the marginal social return plus marginal profits of the firm she deviates to, and lose that corresponding value from the firm she supports in equilibrium. By concavity of the firms' production functions this would lower her payoff. Finally, note that social investors cannot raise their required return r_i^S at the firm they are assigned to as they would violate the firm's incentive compatibility constraint.

Now, to see that this is the investor optimal equilibrium, we note that if, holding the allocation of commercial capital fixed, the capital allocation of social investors across firms was changed, this would by definition reduce the sum of total social investor returns plus total social welfare. Reducing the amount of commercial capital that any firm i raised would weakly reduce social investor welfare, and strictly so if social investors in this equilibrium all strictly prefer their allocation to the outside option asset, as either it would result in firm contraction, or it would need to be replaced by social capital previously allocated to another firm, both of which reduce the sum of investors' welfare by more than r^C by construction. Finally, if $r_i^S = 0$ then increasing the amount of commercial capital held by any firm i is not feasible, as it would result in a social investor earning a negative return. And if $r_i^S > 0$ then by construction we have $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) = r^C$ for all firms that receive social capital, and therefore raising the level of commercial capital would not increase social investors' welfare.

Proof of Proposition 4

Consider the unique equilibrium of the model with values-aligned investors in which no investor utilizes a scale-contingency. In this setting, each firm supported by a social investor operates at the scale $\bar{k}_i$ such that $\pi_i'\left(\bar{k}_i\right) = r_i$. Now consider the case where there are two firms i and j in equilibrium that are both supported by social investors and for whom $\frac{w_i\left(\bar{k}_i\right)}{\bar{k}_i} > \frac{w_j\left(\bar{k}_j\right)}{\bar{k}_j}$ and $w_i'\left(\bar{k}_i\right) < w_j'\left(\bar{k}_j\right)$. The former inequality implies that the equilibrium cost of capital will be lower for firm i, i.e. $r_i < r_j$, and the latter implies that the marginal social value of expanding firm j is higher than for expanding firm i. Now, consider the an arbitrary social investor who supports firm i, and suppose that this investor were to deviate in the following way. Firm i is offered ε less capital, a scale-contingent contract of $\bar{k}_i - \varepsilon$, and charged $\pi_i\left(\bar{k}\right) - \pi_i\left(\bar{k} - \varepsilon\right)$ less. Firm j is offered ε more capital, a scale contingent contract of $\bar{k}_j + \varepsilon$, and charged $\pi_j\left(\bar{k} + \varepsilon\right) - \pi_j\left(\bar{k}\right)$ more. Both firms would accept these contracts, as they do not change the entrepreneurs' share of the profits. And for sufficiently small ε, the deviation would result in higher social welfare as $w_i'\left(\bar{k}_i\right) < w_j'\left(\bar{k}_j\right)$, and increased profit for the investor as $r_i < r_j$.

Proof of Proposition 5

Consider the case where social investors are impact-aligned, and in equilibrium each investor has a strict preference to support some firm i rather than investing in the outside option asset. And now take one such firm i that receives social capital. Let $\bar{k}_i$ be its equilibrium scale. Modify firm i's profitability to a new function $\tilde{\pi}$ in the following way.

$$\frac{d}{dk}\tilde{\pi}\left(k\right) = \begin{cases} \max\left\{\frac{d}{dk}\pi\left(k\right), r^C\right\} & \text{if } k \leq \bar{k}_i \\ \frac{d}{dk}\pi\left(k\right) & \text{else} \end{cases}$$

This modification increases the profitability of firm i so that it can attract commercial capital up to its former equilibrium scale $\bar{k}_i$, and holds fixed the marginal profitability of firm i at all higher scales. Impact-aligned social investors now recognize that their investment up to $\bar{k}_i$ in equilibrium has no impact. Thus the full mass of impact-aligned social investors who used to support firm i up to $\bar{k}_i$ are now free to allocate their capital elsewhere (potentially in part by increasing firm i's scale beyond $\bar{k}_i$), increasing social welfare.

A.5 Section 5.3 and Proof Sketch of Negative Assortative Matching

The welfare-optimal equilibrium can be established following the construction in SectionA.2, where the process is first followed for social investors with $\theta_i = \theta^1$ and then is repeated for each group of social investors with progressively higher levels of altruism. However, to establish that the equilibrium depicted in Figure 6 is indeed welfare-optimal, it is instructive to outline a different method for constructing an equilibrium in this model.

Consider the following process.

Define $\sigma(i)$ to be any ordering over all impact-aligned social investors $i \in S^1 \cup \cdots \cup S^n$.

At step 1, social investor $\sigma^{-1}(1)$ is called to support an entrepreneur. If she chooses an entrepreneur i with profits $\pi_i \geq r^C$, or the outside option asset, assign her a price of $r_i = r^C$, and a social value of $w_i' = 0$. Else assign her a price of $r_i = \pi_i$ and a social value of $w_i' = w_i$. Social investor $\sigma^{-1}(1)$ chooses the entrepreneur i that maximizes $r_i + \theta_{\sigma^{-1}(1)} w_i'$.

At step k, social investor $\sigma^{-1}(k)$ is called on to support an entrepreneur. She chooses an entrepreneur to support according to the same process, excluding any entrepreneur that has been chosen in a previous step. It is straightforward to show that this process results in an equilibrium allocation.

Now, assign every social investor an index that is increasing in their altruism parameter θ_i, so that the least altruistic social investors have the lowest indices. Then the equilibrium depicted in Figure 6 corresponds to the ordering $\sigma(i) = i$. We will demonstrate that the equilibrium arising from any other ordering σ' that results in a different allocation produces lower social welfare than σ.

Take some σ'. Identify two social investors, j and k such that

- $\theta_j > \theta_k$
- $\sigma'(k) = \sigma'(j) + 1$

That is, j is more altruistic than k, but j chooses an investment one step before k in the ordering σ'. If no such pair can be found then $\sigma' = \sigma$. Now consider an alternative ordering σ'', which is the same as σ' except that $\sigma''(j) = \sigma'(k)$ and $\sigma''(k) = \sigma'(j)$ (i.e. j and k are reordered but everything else is preserved). Let a denote the entrepreneur chosen by k under σ' and b denote the entrepreneur chosen by j under σ'. If in the ordering σ'', k chooses a at step $\sigma''(k)$, then the two orderings result in the exact same allocation. Else, under the ordering σ'', at step $\sigma''(k)$, k chooses b. At step $\sigma''(j)$ under the ordering σ'', j chooses an entrepreneur c such that $w_c \geq w_a$, as j is more altruistic than k. In this case, it is straight forward to show that σ'' results in an allocation with weakly higher welfare than does σ'.

Now take σ'' and repeat the above process (i.e. identify mis-ordered pairs of investors and re-order them). Continue to do so until σ'' results in the same allocation as σ. So long as the allocation arising from σ' and from σ are different, it is straightforward to show that at least one transformation resulted in a strict welfare improvement. Therefore σ induces the welfare-optimal equilibrium.

To see that σ is also the investor-optimal equilibrium, we need only demonstrate that the shift from σ' to σ'' described above also weakly improves aggregate investor welfare. If in σ'' k chooses a then the allocation and aggregate investor welfare are the same in σ' and σ'' . Otherwise, k chooses b. This improves k's welfare by revealed preference. If in σ'' j then chooses a, then aggregate investor welfare is unchanged from σ' to σ''.

The remaining case is that under σ'', k chooses b, and j chooses $c \neq a$ with $w_c \geq w_a$ and $\pi_c \leq \pi_a$. We complete the proof by demonstrating that this results in higher aggregate investor welfare than the case where j chose a under σ''. Relative to if j had chosen a under σ'', j's welfare is higher by revealed preference. And all investors who chose before j also have weakly higher welfare, because $w_c \geq w_a$. It remains to show that this also weakly improves the welfare of investors who choose after j under σ''. Relative to the case where j had chosen a, the set of entrepreneurs that these investors can choose from is fixed, except that now c is guaranteed financing, and a remains eligible for financing. Because c has higher social value and lower profits, this can only improve the aggregate welfare of the remaining investors.

A.6 Impact-Aligned Preferences in a Model with A Continuum of Investors and Projects

In this section we discuss the behavior of impact-aligned social investors in a setting with a continuum of investors and projects. In a model with a continuum of projects, social investors cannot influence aggregate social welfare $\int_{\bar{E}} w_j dj$. An intuitive formulation of the utility of impact-aligned social preference – which would not conform with the impact-aligned behavior in our analysis – would be

$$r_i + \theta \int_{\bar{E}} w_j dj \tag{4}$$

Because a single investor cannot influence aggregate social welfare, social investors with the above preferences would single-mindedly optimize their financial return.[32] Nevertheless, the behavior of impact-aligned investors that we analyze can also persist in a continuum model.

Formally, we model the preferences of impact-aligned social investors as arising from the limit of

[32]Indeed, this aligns with Pastor et al. (2020), which considers social investors who have preferences for aggregate social value and concludes that this preference does not influence investor behavior.

a sequence of discrete models, each of which has a finite but increasing number of projects n that can be financed. In each of these models we assume that impact-aligned social investors have utility

$$\frac{1}{n}\left(r_i + \theta \sum_{j \in \bar{E}} w_j\right). \tag{5}$$

As the number of projects financed increases, the contribution of investor i to social welfare *as a fraction of total social welfare* vanishes, yet so does the amount that she values her own financial return. These preferences might be understood to represent the fact that an impact-aligned social investor places the same relative value on her own financial return and the welfare of a fixed set of others no matter how large is the set of total financed projects.

In contrast, the social preferences represented by Equation 4 can be understood as arising from the continuous limit of the same set of discrete models, in which social investor preferences are

$$r_i + \frac{1}{n}\theta \sum_{j \in \bar{E}} w_j.$$

This would correspond to the idea that relative to her own financial return, a social investor places less value on the welfare of a fixed set of others, as the set of all financed projects grows.