Asset Pricing with Optimal Under-Diversification [*]

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Abstract

We study sources and implications of undiversified portfolios in a production economy with financial frictions. Households take concentrated positions in a single firm exposed to idiosyncratic shocks because managerial effort requires equity stakes, and because investors gain private benefits from concentrated holdings. Matching data on returns and portfolios, we find that the marginal investor optimally holds 45% of their portfolio in a single firm, incentivizing managerial effort that accounts for 4% of aggregate output. Investors derive control benefits equivalent to 3% points of excess return, rationalizing low observed returns on undiversified holdings in the data. A counterfactual world of full diversification would feature higher risk free rates, lower risk premiums on fully diversified and concentrated assets, less capital accumulation, yet higher consumption and welfare. Exposure to undiversified firm risk can explain approximately 40% of the level and 20% of the volatility of the equity premium, and generates substantial wealth inequality. A targeted subsidy that decreases diversification improves welfare by increasing managerial effort and reducing financial frictions.

JEL: G12, G51, G32, E44, E21

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1 Introduction

It is a well known fact that the wealthiest households hold large quantities of undiversified assets, mainly due to ownership of private business equity. Based on the 2019 Survey of Consumer Finances, the top 10% wealthiest U.S. households own close to 90% of all (public and private) business equity, and almost 50% of these assets are privately held. Using tax returns, Smith, Zidar, and Zwick (2022) confirm that "pass-through" businesses are the largest category of income for the wealthy. Bach, Calvet, and Sodini (2020) and Fagereng, Guiso, Malacrino, and Pistaferri (2020) find large holdings of private business equity among the wealthy using administrative data on household portfolios for Sweden and Norway, respectively. On the whole, these data strongly suggest that the wealthiest households, who are likely the marginal investors in corporate equity, have large undiversified holdings and thus are exposed to substantial amounts of idiosyncratic business risk. A natural question is why investors choose such high exposure to diversifiable risk, and what the implications are for the macroeconomy and asset prices.

This paper uses a production-based asset pricing model to quantify underlying economic forces that can explain the high level of idiosyncratic risk exposure in portfolios. We then study key implications of this lack of diversification for macro and asset pricing in our calibrated model. First, how are asset prices and real outcomes affected by undiversified investors? How would interest rates, risk premiums, investment, and consumption differ in a counterfactual world of full diversification? Second, can a model with realistic levels of non-diversification help to explain aggregate asset prices? Third, can we reconcile the risk-return trade-off for fully diversified and fully concentrated equity positions within the same model? And finally, is the level of portfolio concentration we capture in the model socially optimal, or would more or less diversification lead to greater social welfare?

To create scope for endogenous under-diversification in our model, we include two mechanisms emphasized in the finance literature. First, in addition to capital and labor, we include a third factor of production, managerial effort in the spirit of Lucas (1978). We assume that supplying this effort requires undiversified positions, capturing the idea that managers must be incentivized by having "skin in the game." Second, investors in our model potentially gain private benefits of control from owning a large stake in an individual business. We infer the quantitative importance of both channels by carefully matching our model to aggregate data on portfolios and returns. To match the large share of idiosyncratic risk in investor portfolios, our estimation requires that roughly 4% of aggregate output are paid to the factor of production that compensates managerial effort. At the same time, we estimate a benefit of private control that compensates investors for 3% points of excess return. In other words, in a counterfactual world without this private benefit, investors would demand 3% points higher excess returns on fully concentrated equity stakes. Jointly, both channels cause the marginal investor in our model to hold 45% of its portfolio in a single firm, which in turn implies that 36% of this portfolio's return risk is due to idiosyncratic shocks.

Our second set of results quantifies the impact of idiosyncratic exposure on aggregate outcomes. Going to a counterfactual world of full diversification by turning off both sources of under-diversification, would lead to substantially higher risk free interest rates and lower excess returns. The risk free rate is 6% points higher and the market risk premium is 2.5% points lower in the full diversification case. The volatility of excess returns drops from 17% to 14%. Thus, we find that lack of diversification can explain approximately 40% of the market excess return and close to 20% of its volatility. On the real side, reduced idiosyncratic risk leads to a smaller precautionary savings motive, resulting in an economy with a 4% smaller capital stock. At the same time, since managerial effort no longer needs to be incentivized through equity stakes, greater effective labor input leads to 2% higher aggregate consumption. Because of higher discount rates, household wealth would shrink by 18%. Yet, household welfare would be 4.15% higher due to much reduced idiosyncratic risk exposure.

The difference between the market equity premium and the excess return earned on undiversified business stakes is the key data moment that allows our model to identify both sources of under-diversification separately. Recent estimates by Xavier (2021) and Smith, Zidar, and Zwick (2022) peg the excess return on concentrated business holdings at around 13%. Given a market risk premium of 6%, our model needs to match this 7% difference, which we interpret as the net "markup" over the fully diversified return required to compensate investors for idiosyncratic risk. If managerial incentives were the only source of non-diversification, the model that matches the 6% market equity premium would predict a higher difference of 10%. Put differently, the required return on concentrated holdings would be much higher than in the data. Conversely, if investors only held concentrated positions to gain utility benefits, the model would predict an almost zero difference – much lower than the data. Only the model with both forces can match asset prices as well as the extent of idiosyncratic risk in data portfolios at the same time.

Finally, we investigate whether investors' privately optimal diversification choice in the calibrated model is also welfare maximizing. The model features an externality: when investors choose which fraction of their portfolio to invest in a single firm, they also change the stochastic discount factor of this firm. A more concentrated ownership stake implies a firm SDF that more heavily loads on idiosyncratic firm risk. Thus, by changing their degree of diversification, investors affect the investment and capital structure choices of firms without internalizing these effects. As an analytical tool, we introduce a tax on managerial compensation that introduces a wedge in investors' optimal choice of portfolio concentration. Welfare in the calibrated model is maximized at a 16% subsidy, causing 2% less diversification. At the optimum, slightly smaller firms face less severe financial frictions and benefit from higher managerial effort, offsetting the costs of higher idiosyncratic risk exposure. In a version of the model without financial frictions, this finding reverses, and welfare is maximized at a 25% tax that penalizes under-diversification.

A key contribution of our paper is to provide a tractable and transparent way of quantifying the sources and consequences of imperfect diversification. In our model, firms are subject to idiosyncratic shocks to capital quality. Households hold the equity of firms in part through a perfectly diversified "stock market" fund, and in part through concentrated equity stakes. Households are further subject to idiosyncratic human capital shocks that may be correlated with their business exposure – capturing the fact that most entrepreneurs work at their own firm. Despite agents facing uninsurable idiosyncratic risks, we keep this framework tractable enough to obtain a representative investor that retains partial exposure to undiversified firm risk. This numerical tractability does not come at the price of an unrealistic wealth distribution. We construct the model to neutralize effects of the wealth distribution for aggregate asset prices – only aggregate wealth matters. However, we generate a substantial amount of wealth inequality through differences in individual portfolio returns.

Related Literature. This paper connects the literature on lack of diversification with the quantitative macro-finance literature that explains aggregate asset prices through time-varying exposure to idiosyncratic risks. Following the classic work of Levy (1978) and Merton (1987), a large literature in finance has studied the sources and consequences of imperfect portfolio diversification. Uppal and Wang (2003) and Bhamra and Uppal (2019), among others, explain underdiversification as result of biases and informational limitations of investors. Others, such as Chen, Miao, and Wang (2010), Chen and Strebulaev (2018), and Iachan, Silva, and Zi (2022) study entrepreneurs with limited ability to diversify the risk of their firms due to market incompleteness, in the spirit of Heaton and Lucas (2000). Our model views retention of idiosyncratic risk as a fundamental aspect of the economy's production technology: incentives of managers and owners depend on undiversified exposure to firm outcomes, a feature of optimal contracts in all canonical compensation models.[1] Yet at the same time, we capture the pass-through of idiosyncratic risk to household-investor portfolios, compatible with the fact that the wealthiest equity investors hold undiversified stakes in their businesses. As result, the stochastic discount factor in our model is volatile due to unsystematic risk exposure, as argued by Dello Preite, Uppal, Zaffaroni, and Zviadadze (2023).

We embed this structure of undiversified household-entrepreneurs in a quantitative asset pricing model. Most prior studies that embed idiosyncratic risk in a production economy focus on the effects of labor income risk, such as Storesletten, Telmer, and Yaron (2007), Gomes and Michaelides (2007), and Favilukis (2013). Unlike those papers, we retain the tractability of Constantinides and Duffie (1996), applied in endowment economies by Constantinides and Ghosh (2017) and Schmidt (2015), among others. We expand the Constantinides-Duffie aggregation approach to general equilibrium with capital accumulation and financial frictions extending the method developed in Diamond and Landvoigt (2022). We advance on these prior papers by considering both idiosyncratic *labor and firm risk*, and by allowing these risks to be correlated at the micro level.[2]

We match the equity premium, the Sharpe ratio of market returns, riskfree rate mean and volatility, as well as basic macro and capital structure moments, in a production economy with financially constrained firms. The main quantitative force is counteryclical idiosyncratic risk exposure that loads on economic crises. In particular, we follow the literature on asset pricing with rare disasters (Barro (2009), Gourio (2012), Gabaix (2012), and Wachter (2013), among

[1] See Edmans and Gabaix (2016) for a comprehensive review.

[2] Our framework includes counteryclical left-skewness in labor income risk as documented by Guvenen, Ozkan, and Song (2014) and as implemented in Catherine (2021). We find that such countercylical labor risk alone is insufficient to generate realistic asset pricing moments. This suggests that the quantitative success of Storesletten, Telmer, and Yaron (2007) and Favilukis (2013) hinges on interactions of countercyclical labor risk with other model elements, such limited participation or as ex-ante heterogeneity in preferences or skill endowments, which are absent from our model.

others) in generating large fluctuations in discount rates for equity. Unlike these prior papers, our model features endogenous exposure of the stochastic discount factor to idiosyncratic risk. Since this new amplification mechanism does a lot of the work, the disasters in our calibrated model are on average shorter and have a smaller permanent productivity component than in the prior literature.[3] At the same time, our approach is compatible with recent evidence that risk premia are primarily due to tail risk events, such as in Schreindorfer (2020).

The model can match asset prices with a plausible corporate capital structure, because it includes financial frictions. Several related papers focus on the interaction of equity returns and corporate financial frictions in general equilibrium, such as Gourio (2013), Favilukis, Lin, and Zhao (2020), Gomes and Schmid (2021), and Smirnyagin and Tsyvinski (2022). We abstract from corporate credit risk and heterogeneity in firm financing patterns. Yet we find that equity issuance costs are they key feature that allows the model to generate volatile equity returns without overstating the volatility of dividends, thus helping to resolve the excess volatility puzzle.

We assume that incentives for effort hinge on undiversified exposure to firm outcomes, which has been documented in numerous papers starting with the seminal studies by Morck, Shleifer, and Vishny (1988) and Ofek and Yermack (2000). Rather than embed optimal contracts in our model, we capture this robust empirical feature of executive compensation by directly tying managerial effort in the production function to under-diversification.[4] The key cost of effort in our model is loss of diversification for the risk averse investor/manager (see Jin (2002) for a similar approach). We also allow for a utility cost of effort as in Edmans, Gabaix, and Landier (2008), yet when matching model to data we find that a net benefit is needed to match the return on undiversified holdings.[5]

By focusing on lack of diversification and exposure to business risk, we connect to the literature that studies the consequences of heterogeneous portfolios and returns for the wealth

[3]Less severe disasters enable us to match model simulations including disaster realizations to the post-WWI sample that includes the Great Depression. This calibration approach provides additional empirical discipline since it avoids treating disasters as a pure "peso problem." The common approach in the literature is to match model simulations conditional on no disaster realizations to the post-WWII sample.

[4]Recent papers that directly embed optimal compensation contracts for managers in asset pricing models are Ai, Kiku, Li, and Tong (2021), Ai and Bhandari (2021), and Ai, Kiku, and Li (2022). Our framework is less directly tied to optimal compensation of managers and rather targets idiosyncratic portfolio risk at the household-investor level.

[5]This is consistent with findings in Fehr, Herz, and Wilkening (2013) and Hurst and Pugsley (2011, 2015) that business owners derive benefits from being in control.

distribution, such Roussanov (2010), Hubmer, Krusell, and Smith (2021), Fagereng, Holm, Moll, and Natvik (2019), Xavier (2021), Catherine (2021), and Greenwald, Leombroni, Lustig, and Van Nieuwerburgh (2021). Relative to this literature, we endogenize returns to aggregate and idiosyncratic risks within the same model. While we do not target the wealth distribution, our framework is able to generate substantial wealth inequality.

By pricing both aggregate and idiosyncratic returns in a unified framework, our paper aims to make progress on empirical puzzles concerned with seemingly low compensation received by investors for taking on idiosyncratic risk, first stated by Moskowitz and Vissing-Jørgensen (2002) and since studied by Kartashova (2014), Xavier (2021), and Boar, Gorea, and Midrigan (2022).

The rest of the paper is structured as follows. Section 2 outlines stylized facts about the amount of idiosyncratic risk in investors' portfolios. Section 3 presents the model. Section 4 explains how the model is solved and calibrated. Section 5 discusses the results, and Section 6 concludes.

2 Motivating Facts

Private Business Equity. Many previous papers have documented that wealthy households have a large fraction of their assets invested in private business equity, such as Saez and Zucman (2016) and Smith, Zidar, and Zwick (2022). We compute the private equity share from the most recent wave of the SCF, shown in the left panel of Figure 1. Blue bars show the fraction of total corporate equity owned by households in each decile of the wealth distribution, with scale on the left axis. As is well known, close to 90% of all business equity (public and private) is owned by the top decile. The solid orange line, with scale on the right, shows the share of equity within each decile that is invested in private businesses. In the top decile, almost 50% of the equity portfolio is private. The dashed black line displays on the same scale the share of publicly traded equity invested at the respondent's own employer. This share is close to 10% for the top decile.

The right panel of Figure 1 performs a similar analysis for wage income. The blue bars in this graph display the fraction of total wage income accruing to households in each decile, with scale on the left. The labor income distribution is much less concentrated than equity

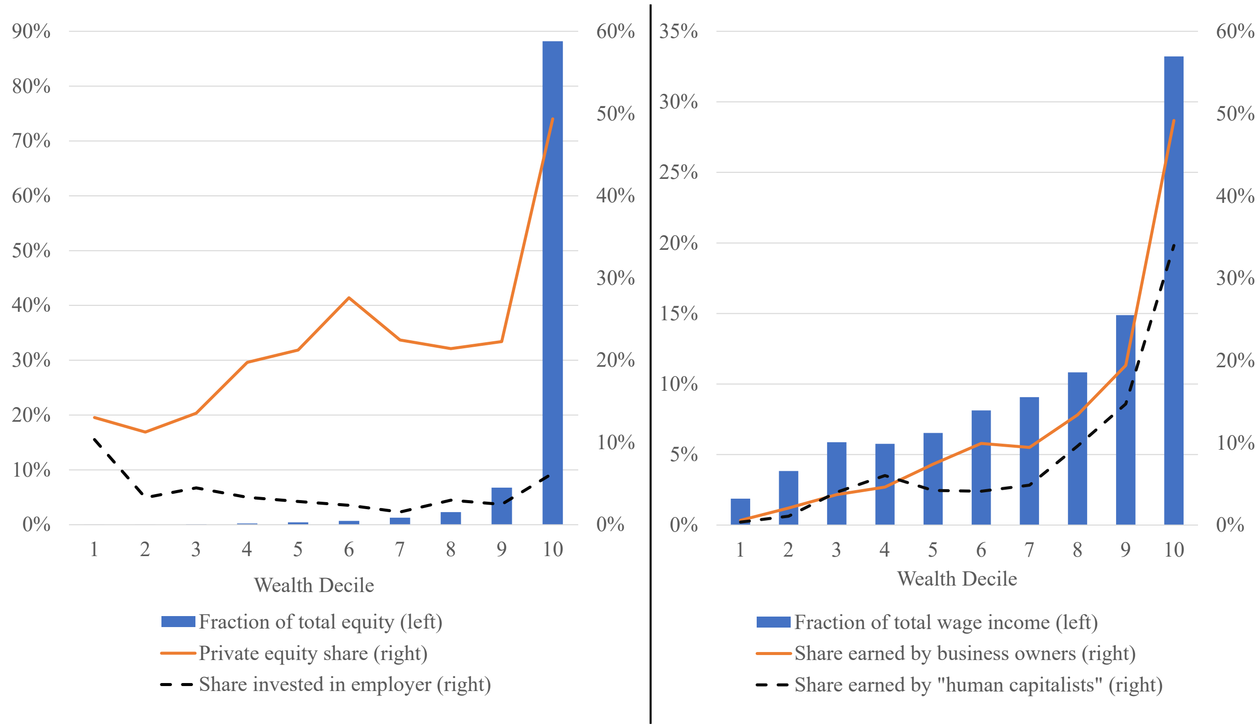


Figure 1: Private Equity and Business Wage Income Shares

ownership, with the top decile only receiving one third of income. The solid orange line shows the share of wage income in each decile earned by survey respondents that own a business.[6] The share of wage income accruing to business owners is large and increasing by wealth decile, with close to 50% of all income in the top decile going to business owners. The dashed black line calculates the share of income earned by "human capitalists" (Eisfeldt, Falato, and Xiaolan (2022)), which are households that own some stock of their publicly traded employer. They are the same survey respondents that reported the amount invested in their employer which is plotted by the dashed black line in the left panel of the figure.

We aggregate these shares into two numbers that have direct counterparts in our structural model. First, we compute the total share of equity held as an undiverified position by averaging the private equity share (solid orange line) across deciles using as weights the fraction of equity owned (blue bars). The resulting total share is 46.1%. When matching our model to the data, a key parameter we identify is the share of the equity portfolio of the representative

[6] The data are inconclusive which fraction of income is actually earned at the own business. For example, households could have a full-time job as an employee at a firm they do not own, but run their own side business in addition. In that case, these households are business owners, but earn most of their income elsewhere. Smith, Yagan, Zidar, and Zwick (2019) find that business owners derive significant human capital compensation by working at their businesses.

Undiversified Portfolio Share	Excess Return	Idiosyncratic Variance Share
Market $\tilde{\eta} = 0$	6.4%	0%
Portfolio $\tilde{\eta} = \eta$	$(1 - \eta)6.4\% + \eta 13.1\%$	35%
Fully Private $\tilde{\eta} = 1$	13.1%	76%

Table 1: Idiosyncratic Excess Returns and Variance Shares

household invested in a single firm. The 46.1% are likely an upper bound for this parameter, since some private business equity positions may be partially diversified – for example, some wealthy households own multiple business in different locations of industries, or they might have invested in partially diversified private equity funds.

Returns and Risk Shares. The second set of facts that serve as inputs for our model are the risk premiums earned on portfolios with different degrees of diversification, as well the relative contribution of aggregate and idiosyncratic risk to the variance of these returns. Foreshadowing the model's notation, we denote by η the fraction of the representative investor's portfolio invested in a single firm. In equilibrium, the stochastic discount factor will therefore have η-exposure to idiosyncratic firm risk. We will use this SDF to price within the model portfolios with different degrees of diversification; we will denote the share of these hypothetical portfolios invested in a single firm $\tilde{\eta}$. By definition, the equilibrium portfolio held by the representative investor has $\tilde{\eta} = \eta$. The first column of Table 1 shows observed excess returns on the fully diversified "market" portfolio ($\tilde{\eta} = 0$) as well as the fully undiversified "private" portfolio ($\tilde{\eta} = 1$). The market excess return of 6.4% is computed from CRSP for the 1990-2019 sample. The excess return earned on the private portfolio of 13.1% is taken from Xavier (2021) who computes the return self-reported business owners achieve in the SCF over the same period. The price of idiosyncratic risk in the model will have an approximately linear slope; thus we know the excess return on the equilibrium portfolio must be the convex combination.

The second column of Table 1 shows the fraction of the variance of excess returns stemming from idiosyncratic risk. By definition, the market portfolio has zero exposure to idiosyncratic risk. We compute variance shares for the other two portfolios using data from Bach, Calvet, and Sodini (2020). Using Swedish administrative data, Bach et al. (2020) compute the excess return on gross wealth for households in different quantiles of the wealth distribution. For each quantile, they also report the idiosyncratic variance share. Using additional data provided by the paper's replication kit, we can aggregate their calculations to a 35% idiosyncratic variance share of the wealth-weighted average investor – this is the exposure to idiosyncratic risk of the equilibrium portfolio in our model. We similarly compute the idiosyncratic variance share of the private portfolio as 76%, implying that 24% return variance of that portfolio is due to aggregate risk. The numbers in Table 1 are the key estimation targets for the model.

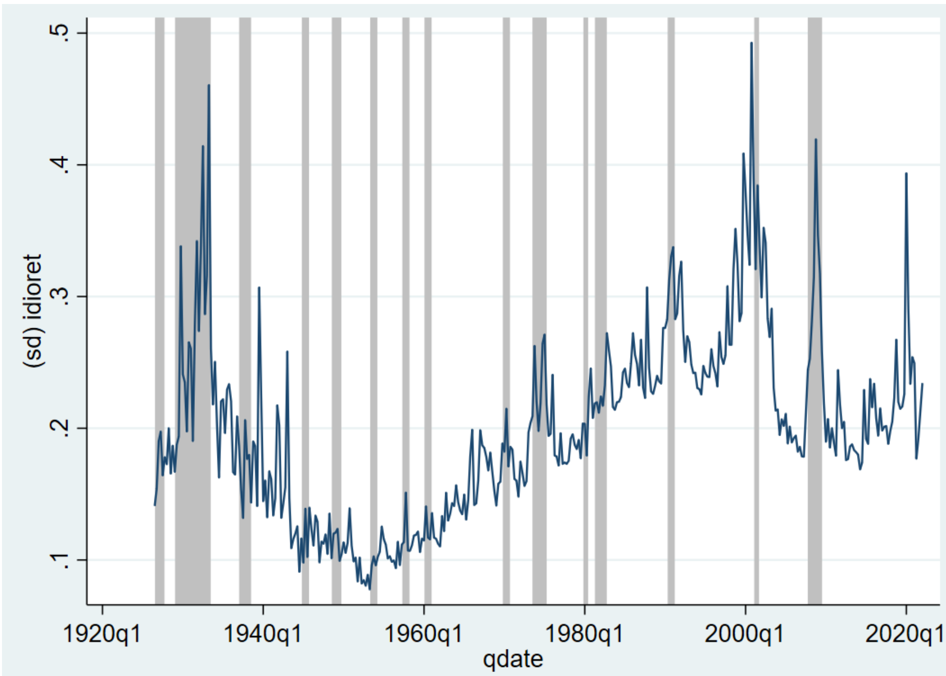


Figure 2: Cross-sectional Dispersion of Idiosyncratic Returns

Time-varying Idiosyncratic Risk. A key feature of the model is time-varying idiosyncratic risk. Fluctuations in higher order moments of firm-level productivity or demand shocks are well-documented.[7] Since our focus in this paper is on the asset pricing consequences of these shocks, we calculate cross-sectional dispersion of idiosyncratic stock returns. Specifically, we compute idiosyncratic returns each quarter as residuals from regressing individual stock returns on time and firm fixed effects. In Figure 2 we plot the standard deviation of residuals. We observe large spikes in the measure of firm risk during the Great Depression, the dot.com bust, and the Great Recession. The latter two spikes are short-lived, while idiosyncratic risk stays

[7] Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018) and Kehrig (2015) estimate large countercylical increases in productivity dispersion. Salgado, Guvenen, and Bloom (2020) estimate substantial left-skewness of productivity and sales during downturns.

persistently elevated during the depression. This motivates our assumption that idiosyncratic risk rises significantly and persistently during rare disasters.

3 Model

The economy consists of a unit measure of households and a unit measure of firms. Households are part of a one-dimensional continuum, indexed by i. Due to the partially concentrated ownership structure of firm equity, each household i is associated with firm i. The exact nature of the firm ownership will be explained in Section 3.3.1.

3.1 Technology

All firms produce the single consumption good with a Cobb-Douglas technology. The production function of a firm with K_t units of capital that hires L_t units of labor and O_t units of managerial effort is

$$F(K_t, L_t, O_t) = A_t K_t^{1-\alpha_L} (Z_t L_t)^{\alpha_L - \alpha_O} (Z_t O_t)^{\alpha_O}, \tag{1}$$

where A_t is total factor productivity (TFP) and Z_t is labor-augmenting productivity. The total labor share in the economy is α_L, with α_O governing the importance of managerial effort. The only source of aggregate productivity fluctuations are rare disasters. The random variable $d_t \in \{0, 1\}$ indicates whether the economy experiences a disaster, $d_t = 1$, or not, $d_t = 0$. It follows a two-state Markov process with transition matrix

$$\Pi_d = \begin{bmatrix} 1 - \pi_d & \pi_d \\ 1 - \pi_s & \pi_s \end{bmatrix},$$

where π_d is the probability of entering into, and π_s is the probability of staying in, the disaster state, respectively. TFP is conditional on the disaster state; it is constant in normal times but experiences negative shocks during disasters:

$$A_t = \bar{A} \exp(-d_t \zeta_t), \tag{2}$$

where $\bar{A}$ is TFP in normal times and ζ_t is the time-varying disaster severity following an AR(1) process

$$\zeta_t = (1-\rho_\zeta)\mu_\zeta + \rho_\zeta \zeta_{t-1} + \epsilon_t^\zeta,$$

with mean μ_ζ, persistence ρ_ζ, and zero mean Gaussian innovations ϵ_t^ζ that have variance σ_ζ^2.[8] Labor-augmenting productivity grows at constant rate $\bar{g}$ and experiences negative i.i.d. growth shocks during disasters

$$Z_t = Z_{t-1}\exp(\bar{g} - \zeta^p d_t \zeta_t). \tag{3}$$

The parameter ζ^p in equation (3) governs the permanent component of rare disasters. Taken together, equations (2) and (3) imply that productivity follows a deterministic growth trend in non-disaster states, yet disaster severity ζ_t always fluctuates. When a disaster occurs, TFP declines by $\exp(\zeta_t)$ and permanent productivity by $\exp(\zeta^p \zeta_t)$.

Firms further have access to an investment technology that produces I_t units of new capital per

$$I_t + \Phi\left(\frac{I_t}{K_t}\right) K_t$$

units of the consumption good used as input, where the adjustment cost function Φ is convex in the investment rate.

3.2 Firms

Firms maximize the present value of dividends paid to their shareholders. Capital is traded among firms in a perfectly liquid market at price q_t. Firms can also issue one-period risk free bonds to households at price p_t.

Firms are subject to financial frictions and idiosyncratic capital quality shocks. The financial frictions consist of a borrowing constraint and costly equity issuance. First, firms' ability to issue debt is limited by a collateralized borrowing constraint. Second, firms are expected to pay out a fraction of their net worth to households as dividend each period. Firms that deviate from this target incur a convex cost; this means that large dividend payouts and cutting dividends/issuing new equity are costly.

Idiosyncratic capital quality shocks $\epsilon_{t,i}^F$ are distributed identically and independently across

[8] Our approach to modeling disasters builds on Gabaix (2012).

firms and have mean $\mathrm{E}[\epsilon_{t,i}^F] = 1$. They are drawn from marginal distribution F_t^F whose parameters may vary over time. In particular, the standard deviation $\sigma_{E,t} = \mathrm{Var}_t(\epsilon_{t,i}^F)^{1/2}$ varies with the disaster state of the economy

$$\sigma_{E,t} = \bar{\sigma}_E + d_t \zeta^\sigma.$$

It is equal to $\bar{\sigma}_E$ in normal times and jumps by ζ^σ during disasters.

Firm profits net of capital depreciation and interest expenses are taxed at rate τ. Tax deductibility of interest creates a motive for leverage. In our model, firm debt is risk-free. To match the strength of the tax shield in the data where borrowing is risky and firms pay a spread, we define the interest rate for tax purposes as r^{int} such that the tax shield is given by $\tau^{int} = \tau \times r^{int}$.

The decisions of firms within each period can be divided into two stages, a production and a portfolio stage:

1. **Production:** Given capital after realization of the time t capital quality shock, $\bar{K}_{t,i}$, firms demand labor $L_{t,i}$ at competitive real wage w_t, managerial effort $O_{t,i}$ at fee f_t, and produce the consumption good. Further, firms choose investment $I_{t,i}$, which they sell in the capital market.

2. **Portfolio:** Firms sell their capital after depreciation and repay old debt $B_{t,i}$, pay fraction ξ_0 of their net worth as dividend to shareholders, raise new equity $X_{t,i}$ from shareholders, purchase capital $K_{t+1,i}$ for next period, and issue new debt $B_{t+1,i}$.

We first define the firm's profit at the production stage:

$$\Pi_{t,i} = F(\bar{K}_{t,i}, L_{t,i}, O_{t,i}) - w_t L_{t,i} - f_t O_{t,i} - I_{t,i} - \Phi\left(\frac{I_{t,i}}{\bar{K}_{t,i}}\right)\bar{K}_{t,i} + q_t I_{t,i}. \tag{4}$$

Given this profit, the firm's after-tax net worth at the portfolio stage is

$$N_{t,i} = (1-\tau)\Pi_{t,i} + (1-(1-\tau)\delta)q_t\bar{K}_{t,i} - (1-\tau^{int})B_{t,i}, \tag{5}$$

where δ is the depreciation rate of capital.

To raise new equity $X_{i,t}$, shareholders need to pay

$$X_{i,t} + \Xi\left(\frac{X_{t,i}}{N_{t,i}}\right) N_{t,i},$$

where the last term is the convex equity issuance cost.

Finally, a borrowing constraint limits debt issuance for next period to a fraction θ_t of capital purchases

$$B_{t+1,i} \leq \theta_t K_{t+1,i}. \tag{6}$$

Under these assumptions, we can write the firm problem at the portfolio stage recursively as[9]

$$V^F(N_{t,i}, \mathcal{Z}_t) = \max_{K_{t+1,i}, B_{t+1,i}, X_{t,i}} \xi_0 N_{t,i} - X_{t,i} - \Xi\left(\frac{X_{t,i}}{N_{t,i}}\right) N_{t,i} + \mathrm{E}_t\left[\mathcal{M}^i_{t,t+1} V^F(N_{t+1,i}, \mathcal{Z}_{t+1})\right] \tag{7}$$

subject to the budget constraint

$$(1-\xi_0)N_{t,i} + X_{t,i} \geq q_t K_{t+1,i} - p_t B_{t+1,i}, \tag{8}$$

the transition law for net worth implied by (4) and (5)[10] and the borrowing constraint in (6). The firm's stochastic discount factor $\mathcal{M}^i_{t,t+1}$ in (7) is defined below in equation (19). Due to the undiversified nature of firm ownership, the SDF is firm-specific and uniquely determined as the SDF that aligns the optimal choices of the firm for capital, debt, and equity issuance with those of its imperfectly diversified shareholder. Importantly, SDF $\mathcal{M}^i_{t,t+1}$ only depends on the identity of firm i through the realization of the idiosyncratic shock $\epsilon^F_{t+1,i}$. Since the shocks are i.i.d., this guarantees that the firm problem is fully symmetric across firms. In Appendix A.1, we show that under these assumptions, the firm objective is homogeneous of degree one in net worth. We define the ratios $k_{t+1} = K_{t+1,i}/N_{t,i}$, $x_t = X_{t,i}/N_{t,i}$, and $b_{t+1} = B_{t+1,i}/N_{t,i}$. Using these normalized decision variables, we define a value function $v^F(\mathcal{Z}_t) = V^F(N_{t,i}, \mathcal{Z}_t)/N_{t,i}$.

The homogeneity in net worth implies that all firms choose the same ratios k_{t+1}, b_{t+1}, and x_t. The value function per dollar of net worth, $v^F(\mathcal{Z}_t)$, is the same for all firms, since it does

[9] The production problem yields static first-order conditions for labor demand and investment, see Appendix A.1.

[10] This transition law implies that an individual's firm capital $\bar{K}_{t+1,i} = K_{t+1,i}\epsilon^F_{t+1,i}$ i.e. is the product of aggregate capital and the firm's idiosyncratic shock.

not depend on the level of net worth $N_{t,i}$, but only its growth rate

$$n_{t+1,i} = \frac{N_{t+1,i}}{N_{t,i}} = k_{t+1}\epsilon^F_{t+1,i}R^k_{t+1} - (1-\tau^{int})b_{t+1}, \tag{9}$$

where $R^K_{t+1} = (1-\tau)r^K_{t+1} + (1-(1-\tau)\delta)q_{t+1}$. This growth rate in turn only depends on idiosyncratic firm variables is through the i.i.d. shock realizations.

3.2.1 Aggregation of Firms

The growth rate of net worth of firm i is given by (9), where k_{t+1} is the capital-net worth ratio and b_{t+1} is the debt-net worth ratio common across all firms. This implies that the average growth rate of net worth across all firms in $t+1$ is

$$\bar{n}_{t+1} = \mathrm{E}_{\epsilon^F}[n_{t+1,i}] = k_{t+1}R^K_{t+1} - (1-\tau^{int})b_{t+1}, \tag{10}$$

since $\mathrm{E}_{\epsilon^F}[\epsilon^F_{t+1,i}] = 1$. Denote aggregate net worth of firms at the beginning of t as N_t. Then the transition law of N_t is

$$N_{t+1} = N_t\bar{n}_{t+1}. \tag{11}$$

3.3 Households

Households are infinitely-lived and invest in debt and equity issued by firms. They further trade claims to the two types of human capital – labor and managerial effort – in a perfectly competitive market among each other.

3.3.1 Investing in Corporate Equity

Households can only invest in firms through a fund that holds fraction $1-\eta_{t,i}$ of corporate equity in perfectly diversified shares of all firms, while the remaining fraction $\eta_{t,i}$ is invested in a single firm. Each household i invests in a specific fund i with exposure $\eta_{t,i}$ to a single firm that is *newly randomly selected from all firms each period*. In particular, the growth rate of firm equity held by fund i has exposure $\eta_{t,i}$ to the idiosyncratic risk of firm i:

$$n^\eta_{t+1,i} = (1-\eta_{t,i})\bar{n}_{t+1} + \eta_{t,i}n_{t+1,i}.$$

Using the definition of the average growth rate in (10), this can be written as

$$n^{\eta}_{t+1,i} = \left(1 + \eta_{t,i}(\epsilon^{F}_{t+1,i} - 1)\right) k_{t+1} R^{K}_{t+1} - (1 - \tau^{int}) b_{t+1}. \tag{12}$$

Households choose the size of their individual fund in terms of the net worth of the firms held by the fund. They further choose the degree of concentration η_t. The choice of η_t is directly linked to the provision of managerial effort, to be explained in section 3.3.2. For each dollar of net worth held by the fund, households need to pay market price Q_t to buy the shares of the fund. Specifically, if household i chooses fund size $S_{t+1,i}$, then expenditure at t is $S_{t+1,i}Q_t$ and the total payoff at $t+1$ is

$$S_{t+1,i} n^{\eta}_{t+1,i} \left(D_{t+1} + Q_{t+1}\right), \tag{13}$$

where $D_{t+1} = \xi_0 - x_{t+1} - \Xi(x_{t+1})$ is the dividend per dollar of firm net worth paid out by the fund based on (35) in Appendix A.1. This ownership structure has two immediate implications. First, market clearing requires that the aggregate fund size chosen by households is equal to total firm equity,

$$\int S_{t+1,i}\, di = N_t.$$

Second, since households have different levels of wealth and choose different sizes of equity positions that have undiversified components, the firm size distribution needs to be compatible with the demanded distribution of fund sizes by households. Due to the linear homogeneity of the firm problem in net worth, the firm size distribution is irrelevant for aggregate outcomes. Therefore, we assume that the corporate sector is "restructured" each period after the production stage to comply with the size distribution required by household portfolios. Since shocks are i.i.d. and only aggregate firm net worth matters, this assumption is innocuous. The critical assumption needed for aggregation is that each household is randomly assigned to a new firm each period.

3.3.2 Household Problem

At the beginning of the period, households have labor human capital acquired in the last period, $H^{L}_{t,i}$. Households inelastically supply all labor to firms – each unit of human capital acquired in the previous period generates one unit of labor, for which firms pay wage w_t. Tax revenue raised by the government through corporate taxes is paid out to households as part of their labor income,[11] such that the effective wage is $\tilde{w}_t = w_t + [\text{tax rebate}]_t$, where $[\text{tax rebate}]_t$ is the rebated tax revenue per unit of human capital defined below in (28). After supplying labor to firms, households sell their human capital at market price q_t^L.

Household i further decides on consumption $C_{t,i}$, purchases of corporate debt $B_{t+1,i}^H$, how much to invest in the stock fund $S_{t+1,i}$, the degree of stock fund concentration $\eta_{t,i}$, and how much human capital of each type to purchase for next period: $(H_{t+1,i}^L, H_{t+1,i}^O)$.

Households further supply managerial effort, compensated at fee f_t. The supply of managerial effort is proportional to the degree of portfolio concentration $\eta_{t,i}$ so that household i supplies $\eta_{t,i} H_{t+1,i}^O$. The direct connection between managerial effort and non-diversification reflects an ownership and compensation structure that provides incentives for managers to exert effort.[12]

The amount of labor human capital owned by household i is subject to idiosyncratic shocks $\epsilon_{t,i}^H \sim G_t^H$. These shocks have $\mathrm{E}[\epsilon_{t,i}^H] = 1$ and are i.i.d. across households and time. Similar to idiosyncratic firm shocks, the human capital shock distribution depends on the aggregate state. In particular, we follow the empirical literature on cyclicality of labor earnings (Guvenen, Ozkan, and Song (2014)) and assume that Kelly's skewness κ_t^H of the distribution G_t^H becomes negative during disasters.

Recall that households have only imperfectly diversified equity claims, with the payoff of household i's portfolio given by (13). We assume that the human capital shocks of household i are potentially correlated with the capital quality shocks $\epsilon_{t,i}^F$ experienced by the firm that makes up the undiversified part of the household's equity portfolio; mathematically, the two idiosyncratic shocks are jointly distributed $(\epsilon_{t,i}^F, \epsilon_{t,i}^H) \sim G_t$. This assumption reflects the possibility that household i derives part of its labor income from working at firm i.[13]

We further assume that managerial human capital is fully exposed to firm-level capital quality shock $\epsilon_{t,i}^F$. This is consistent with this effort being provided specifically at the firm that the household partially owns.

Household i enjoys consumption $C_{t,i}$ and also receives utility from owning a concentrated

[11] We abstract away from other forms of taxation. The rebate can be interpreted as the result of progressive income taxation. Since tax revenue is a small portion of output, different assumptions regarding its distribution (e.g., the revenue disappears from the economy) have negligible effects on our quantitative results.

[12] This assumption can be derived as the outcome of an underlying moral hazard problem.

[13] To implement the risk in left-skewness of human capital shocks, we model the distribution based on a mixture of normal; see calibration Section C.1.

stake in a firm, leading to the following felicity function

$$u(C_{t,i}, \eta_{t,i}, S_{t+1,i}) = C_{t,i} + \psi S_{t+1,i}\sqrt{\eta_t}.$$

We include the second utility term as an additional source of non-diversification: if $\psi > 0$, households enjoy "being in charge," with the benefit increasing linearly in the size of the firm and concave in the degree of concentration.[14] Intuitively, running a large firm is better than running a small firm, and, once one already has significant control, more control is not worth as much.

Households have recursive preferences over this bundle, with IES $1/\gamma$, risk aversion σ, and discount factor β are

$$U_{t,i} = \left((1-\beta)u(C_{t,i}, \eta_{t,i}, S_{t+1,i})^{1-\gamma} + \beta \mathrm{E}_t \left[U_{t+1,i}^{1-\sigma}\right]^{\frac{1-\gamma}{1-\sigma}}\right)^{1/(1-\gamma)}. \tag{14}$$

Denoting household wealth at the beginning of the period as $W_{t,i}$, the recursive problem is

$$V^H(W_{t,i}, \mathcal{Z}_t) = \max_{\substack{S_{t+1,i}, B^H_{t+1,i}, H^L_{t+1,i},\\ H^O_{t+1,i}, C_{t,i}, \eta_{t,i}}} \left(u(C_{t,i}, \eta_{t,i}, S_{t+1,i})^{1-\gamma} + \beta \mathrm{E}_t \left[V^H(W_{t+1,i}, \mathcal{Z}_{t+1})^{1-\sigma}\right]^{\frac{1-\gamma}{1-\sigma}}\right)^{1/(1-\gamma)} \tag{15}$$

subject to the budget constraint

$$W_{t,i} + f_t \eta_{t,i} H^O_{t+1,i} \geq C_{t,i} + Q_t S_{t+1,i} + p_t B^h_{t+1,i} + q^L_t H^L_{t+1,i} + q^O_t H^O_{t+1,i}, \tag{16}$$

the transition law for wealth

$$W_{t+1,i} = S_{t+1,i} n^{\eta}_{t+1,i} \left(D_{t+1} + Q_{t+1}\right) + B^H_{t+1,i} + (\tilde{w}_{t+1} + q^h_{t+1})\epsilon^H_{t+1,i} H^L_{t+1,i} + q^O_{t+1}\epsilon^F_{t+1,i} H^O_{t+1,i}, \tag{17}$$

$\eta_t \leq 1$ and no-shorting conditions for all assets.

3.3.3 Aggregation of Households

Our assumption on the ability to trade human capital makes the household value function homogeneous in wealth. Therefore, similar to the firm problem, we can define value function

[14] Fehr, Herz, and Wilkening (2013) provide experimental evidence of preference for authority.

$v^H(\mathcal{Z}_t) = V^H(W_{t,i}, \mathcal{Z}_t)/W_{t,i}$, and construct the stochastic discount factor of a representative household. Based on this household SDF, we can define the firm SDF that aligns firm and household choices. The following proposition summarizes these results, with the proof in Appendix A.2.

Proposition 1. *1. The stochastic discount factor of the representative household is*

$$M^i_{t,t+i} = \beta \left(\frac{v^H(\mathcal{Z}_{t+1})}{v^H(\mathcal{Z}_t)} \right)^{1-\gamma} \left(\frac{v^H(\mathcal{Z}_{t+1})}{CE_t} \right)^{\gamma-\sigma} (r^H_{t+1,i})^{-\sigma}, \tag{18}$$

where

$$r^H_{t+1,i} = \frac{W_{t+1,i}}{W_{t,i}}$$

is the return on household wealth, which only depends on household i variables through the realization of $(\epsilon^F_{t+1,i}, \epsilon^h_{t+1,i})$, and

$$CE_t = E_t \left[(v^H(\mathcal{Z}_{t+1}) r^H_{t+1,i})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

is the value function certainty equivalent.

2. The firm stochastic discount factor that equalizes firm and household valuation of corporate equity is

$$\mathcal{M}^i_{t,t+1} = M^i_{t,t+i} \frac{n^\eta_{t+1,i}}{n_{t+1,i}}. \tag{19}$$

Given this firm SDF, the market price of corporate equity is

$$Q_t = \psi\sqrt{\eta_t} + E_t \left[M^i_{t,t+1} n^\eta_{t+1,i} (D_{t+1} + Q_{t+1}) \right] = \psi\sqrt{\eta_t} + E_t \left[\mathcal{M}^i_{t,t+1} n_{t+1,i} v^F(\mathcal{Z}_{t+1}) \right],$$

and $v^F(\mathcal{Z}_t) = D_t + Q_t$.

Part 1 of the proposition constructs the SDF of the representative household; due to the homogeneity of the value function, all households choose the same consumption and portfolio shares out of wealth, and the realized return on wealth only differs in the realization of idiosyncratic shocks. Since all households face the same distribution of idiosyncratic risk, the SDF is symmetric across households. An important implication of this result is that all households choose the same level of portfolio concentration η_t. Part 2 specifies the SDF used in the optimization problem of the firm. Firms are by construction fully exposed to idiosyncratic capital quality shocks $\epsilon^F_{t,i}$, whereas household-shareholders are only exposed to these shocks with fraction η_t of their equity portfolio. The correction $\frac{n^\eta_{t+1,i}}{n_{t+1,i}}$ to the household SDF in the firm problem ensures that all dynamic firm choices – capital, debt, and equity issuance – are optimally determined as if they had been chosen by the household-shareholder directly. Due to the linearity of the firm problem in net worth, it is possible to eliminate the wedge between firm (manager) and shareholder decisions stemming from differential exposure to idiosyncratic risk through this simple linear transformation of the SDF.[15]

3.4 Equilibrium

Market clearing requires

$$\int H^L_{t,i}\, di = L_t, \qquad \text{Labor} \tag{20}$$

$$\int \eta_{t,i} H^O_{t+1,i}\, di = O_t, \qquad \text{Effort} \tag{21}$$

$$\int S_{t+1,i}\, di = N_t, \qquad \text{Firm Equity} \tag{22}$$

$$\int H^L_{t+1,i}\, di = \bar{H}^L, \qquad \text{Human Capital} \tag{23}$$

$$\int H^O_{t+1,i}\, di = \bar{H}^O, \qquad \text{Managerial Capital} \tag{24}$$

$$\int B^H_{t+1,i}\, di = B_{t+1}, \qquad \text{Debt} \tag{25}$$

$$K_{t+1} = (1-\delta)K_t + I_t. \qquad \text{Capital} \tag{26}$$

The aggregate resource constraint for the consumption good is

$$Y_t = C_t + I_t + K_t \Phi\left(\frac{I_t}{K_t}\right) + N_t \Xi\left(\frac{X_t}{N_t}\right). \tag{27}$$

The first market clearing condition in (20) equates labor supply by households with labor demand from firms. Households inelastically supply their complete labor endowment that they purchased in the previous period. Based on the market clearing condition for human capital

[15] Setting up the firm problem to maximize shareholder utility directly would yield the same result. We abstract away from potential agency conflicts between shareholders and managers of the firm.

(23), this is equal to the aggregate human capital endowment $\bar{H}^L$, i.e. in equilibrium labor is $L_t = \bar{H}_L$. Supply of managerial effort, however, depends on the degree of diversification: combining (21) with (24) implies $O_t = \eta_t \bar{H}^O$. The market clearing condition for corporate equity (22) says that the net worth held by households' equity funds needs to equal corporate net worth N_t. Condition (25) equalizes the debt issued by firms with that bought by households. Finally, capital market clearing in (26) requires that the demand for new capital K_{t+1} equals supply, consisting of undepreciated old capital K_t and new investment. The resource constraint (27) clarifies that aggregate output is either spent on consumption, investment, or the two adjustment costs of firms.

Each period, the government raises corporate tax revenue

$$T_t = \tau K_t (r_t^K - \delta) - \tau^{int} B_t.$$

This revenue is rebated to households as part of their labor income, i.e.

$$[\text{tax rebate}]_t = \frac{T_t}{\bar{H}}. \tag{28}$$

3.5 Discussion: Asset Pricing with Imperfect Diversification

Optimal Portfolio Concentration. Appendix A.2 derives the household first-order condition for the choice of η_t:

$$h_{t+1}^O f_t + \frac{\psi s_{t+1}}{2\sqrt{\eta_t}} = s_{t+1} \mathrm{E}_t \left[M_{t,t+1}^i v^F(\mathcal{Z}_{t+1})(1 - \epsilon_{t+1,i}^F) k_{t+1} R_{t+1}^K \right], \tag{29}$$

where s_{t+1} and h_{t+1}^O time-t household choice variables divided by wealth $W_{t,i}$. The two terms on the left capture the marginal benefits of greater concentration. First, the household earns managerial fee income $h_{t+1}^O f_t$ for each marginal increase in concentration. Second, the household experiences a marginal utility increase of $\frac{\psi s_{t+1}}{2\sqrt{\eta_t}}$ with a rise in η_t. The two benefits must equal the cost on the right-hand side, which is the priced amount of idiosyncratic risk the household takes on by choosing $\eta_t > 0$. Note that $\mathrm{E}_t \left[1 - \epsilon_{t+1,i}^F \right] = 0$; an agent without exposure to idiosyncratic firm risk $\epsilon_{t+1,i}^F$ would therefore not experience any marginal cost and would choose maximal $\eta_t = 1$ i.e. a fully private portfolio. However, the idiosyncratic shock realization $\epsilon_{t+1,i}^F$ appears in (29) inside of the SDF $M^i_{t,t+1}$ (18) through the return on wealth

$$r^H_{t+1,i} = s_{t+1} v^F(\mathcal{Z}_{t+1}) n^\eta_{t+1,i} + (w_{t+1} + q^h_{t+1})\epsilon^H_{t+1,i} h_{t+1} + b^H_{t+1},$$

since net worth growth for firm i, $n^\eta_{t+1,i}$ given by (12) also loads on $\epsilon^F_{t+1,i}$, as long as η_t is positive. Thus, when a household chooses to take a large stake in an individual firm, it causes its wealth to depend on idiosyncratic firm risk, which is costly due to risk aversion. The cost term on the right of (29) is therefore positive for $\eta_t > 0$. Under which conditions would it be optimal for investors to choose a fully diversified portfolio, $\eta_t = 0$? This special case of our model obtains when $\alpha_O = 0$, so that managerial effort is not needed for production and therefore fee income $f_t = 0$, and when $\psi = 0$, i.e. investors do not get utility benefits from holding large stakes of companies. In that case, both terms on the left of (29) would be zero. We will estimate the values of α_O and ψ consistent with the amount of portfolio concentration we observe in the data.

Equity Premium with Idiosyncratic Risk. The degree of exposure to idiosyncratic firm risk in household i's equity position is governed by η_t. The effect of $\eta_t > 0$ can best be seen by inspecting the household first-order condition for equity, derived in Appendix A.2:

$$Q_t = \psi\sqrt{\eta_t} + \mathrm{E}_t\left[M^i_{t,t+1} v^F(\mathcal{Z}_{t+1}) \left(\underbrace{\left(1 + \eta_t(\epsilon^F_{t+1,i} - 1)\right) k_{t+1} R^K_{t+1} - (1-\tau^{int}) b_{t+1}}_{= n^\eta_{t+1,i}} \right) \right]. \qquad (30)$$

The idiosyncratic shock realization $\epsilon^F_{t+1,i}$ appears in (30) both in the priced cash flow and inside of the SDF (18). This immediately implies that fraction η_t of idiosyncratic firm risk is "priced", since it is part of portfolio returns, and it directly affects the price of risk. In fact, even along the model's balanced growth path without aggregate shocks ("steady state"), households demand a risk premium for equity, as can be seen from the steady state version of condition (30).[16]

[16] The steady state condition is

$$Q = \psi\sqrt{\eta} + \beta v^F \mathrm{E}_\epsilon[r^H(\epsilon)^{1-\sigma}]^{\frac{\sigma-\gamma}{1-\sigma}} \left(\mathrm{E}_\epsilon[(1+\eta(\epsilon^F - 1))\, r^H(\epsilon)^{-\sigma}] R^K k - \mathrm{E}_\epsilon[r^H(\epsilon)^{-\sigma}](1-\tau^{int}) b \right),$$

where $\epsilon = (\epsilon^F, \epsilon^H)$ is the vector of idiosyncratic shocks. If we further eliminate idiosyncratic risk, this condition collapses to $Q = \beta\exp(-\gamma\bar{g})(R^K k - (1-\tau^{int})b)$, since in that case the steady state return on household wealth is simply $r^H = e^{\bar{g}}$.

However, uninsurable idiosyncratic risk will generally induce a precautionary savings motive in steady state, affecting prices of all assets, and a risk premium for assets whose cash flows are exposed to the idiosyncratic shocks. Moreover, idiosyncratic firm risk is time-varying and spikes during disasters. Exposure to this risk therefore causes the price of risk to rise during disasters and leads to higher risk premiums for both aggregate and idiosyncratic risk.

The term $\psi\sqrt{\eta_t}$ represents the marginal non-pecuniary benefit of owning a stake in the firm. $\psi > 0$ increases the price of equity and therefore lowers expected returns.

The Public Equity Premium and the Price of Idiosyncratic Risk. The excess return on equilibrium equity portfolios when $\eta_t > 0$ in the model should be understood as risk premium on a mix of diversified and undiversified portfolio components, e.g. a mix of public and private equity positions. While the only equity claim traded in equilibrium has idiosyncratic exposure η_t, within the model we price a full continuum of claims with cash flow exposure $\tilde{\eta}$. Mathematically, we price assets with cash flows

$$\mathcal{P}_{t+1}(\tilde{\eta}) = v^F(\mathcal{Z}_{t+1})\left(\left(1+\tilde{\eta}(\epsilon^F_{t+1,i}-1)\right)k_{t+1}R^K_{t+1} - (1-\tau^{int})b_{t+1}\right)$$

and therefore prices

$$Q_t^{(\tilde{\eta})} = \psi\sqrt{\tilde{\eta}} + \mathrm{E}_t\left[M^i_{t,t+1}\mathcal{P}_{t+1}(\tilde{\eta})\right]. \tag{31}$$

In equation (31), SDF $M^i_{t,t+1}$ always has exposure to idiosyncratic firm shocks of η_t as optimally chosen by investors based on (29). We now use this SDF to price a continuum of hypothetical assets with cash flows indexed by $\tilde{\eta} \in [0, 1]$. The marginal utility benefit $\psi\sqrt{\tilde{\eta}}$ also varies with the hypothetical size of the ownership stake $\tilde{\eta}$. By definition, $Q_t^{(\eta_t)} = Q_t$ from (30), i.e. setting $\tilde{\eta} = \eta_t$ prices the equilibrium portfolio. By the same logic, $Q_t^{(0)}$ is the price of the fully diversified "market" portfolio and $Q_t^{(1)}$ is the price of the fully undiversified "private" portfolio. We can then compute returns on these portfolios as $R_t(\tilde{\eta}) = \mathrm{E}_t\left[\mathcal{P}_{t+1}(\tilde{\eta})\right]/Q_t^{(\tilde{\eta})}$. We will estimate the price of idiosyncratic risk in the model by matching the market return $R_t(0)$ and the private return $R_t(1)$ to their counterparts in the data. These returns in the data are the end points of the line depicted in Figure 3.[17]

Generally, η will affect both slope and level of this line: even the market portfolio with $\tilde{\eta} = 0$

[17] From the data, we only know the end points, but not the shape of the line.

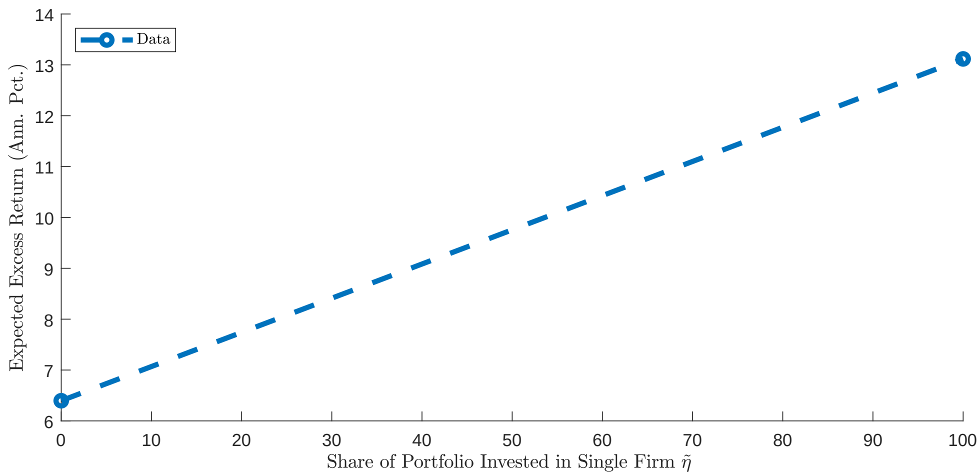


Figure 3: Excess Returns by Exposure $\tilde{\eta}$

is affected by investor exposure to idiosyncratic risk. The key mechanism in the model is that the presence of idiosyncratic firm risks will make the household SDF more volatile, and induce a stronger negative correlation even with diversified firm cash flows $v^F(\mathcal{Z}_{t+1})\bar{n}_{t+1}$.

The Role of Human Capital Shocks. Many previous studies have emphasized the importance of uninsurable labor income risk in explaining high risk premia on stocks. We incorporate this potential source of variation in the price of risk in our model. Relative to the existing literature, we allow for correlated risks in labor income and stock portfolios at micro level, i.e. the realizations of ϵ_t^F and ϵ_t^H may be correlated. This reflects that employees may have equity stakes in the companies they work for (voluntary or because of compensation schemes), and that entrepreneurs have equity stakes in the businesses they own and manage. Our calibration further features correlated time-variation in the dispersion of idiosyncratic firm risks and negative skewness in idiosyncratic labor risk – firm-level capital quality shocks become more dispersed at the same rime that human capital shocks exhibit increase left-skewness.

4 Solution, Calibration and Model Fit

4.1 Solution Method

The model is solved numerically. There are two exogenous and three endogenous state variables – capital stock, firm net worth, and household wealth. Market-clearing conditions allow us to eliminate one state variable – household wealth – when solving the model. The presence of large, non-normal shocks, substantial risk and risk aversion, and an occasionally binding constraint make prices and quantities highly nonlinear functions of the state space. We solve the model globally using transition function iteration (TFI) adapted from Elenev et al. (2021). Equilibrium objects such as state variable transitions, prices, and quantities are represented by linearly interpolated functions on a rectangular state space grid and initialized at their deterministic steady state values. The model is solved when successive iterations produce approximately identical solutions to equilibrium objects, and a long simulation of the model delivers errors in equilibrium conditions below a given tolerance.

To derive means and volatilities of various objects of interest, we simulate many long paths of the model. Most simulated periods are no-disaster states ($d_t = 0$) while the rest are disasters of varying sizes – mild, average, and severe. As we show in Appendix C.1, our disasters are mainly large transitory declines in productivity, with a relatively small permanent component. Compared to the disaster calibrations in Wachter (2013) and Gourio (2012), our disasters less frequent and less severe. This different calibration strategy reflects our view that the U.S. post-WWI (1919-2019) sample contains at least one disaster with the Great Depression, which is not quite severe enough to be considered a disaster based on common calibrations. Our approach entails matching the volatility of consumption growth in model simulations that include average-sized disasters to the 1919-2019 sample in the data. This approach differs from the usual one of matching model output conditional on "no disaster occurrence" to the post-WWII (1954-2019) sample.

To compute impulse responses, we initialize the economy at a particular point in the state space closest to the deterministic steady state, and simulate transitions into a different state implied by the impulse. Subsequently, we let the economy evolve stochastically from the impulse state to the ergodic steady state, simulating many paths of these evolutions and reporting the mean path.

To accommodate pairs of correlated idiosyncratic shocks, we use bivariate Gaussian quadrature, discretizing the support of $(\epsilon_i^F, \epsilon_i^H)$. Allowing for time-varying dispersion $\sigma_{E,t}$ and skewness κ_t^H requires a semi-parametric approach to numerical integration, with details provided in Appendix B.1.

Table 2: Externally Calibrated Parameters

Par	Description	Value	Source
		Aggregate Productivity Risk	
$\bar{g}$	Average productivity growth rate	1.8%	Labor productivity growth (1954-2019)
π^d	Quarterly prob. of disaster	0.62%	Uncond. ann. dis. prob. 3.5% (Wachter, 2013)
π^s	Quarterly prob. of staying in disaster	70%	Expec. disaster length 1 year
ζ^p	Permanent disaster multiple	0.05	Post WWI U.S. disasters
ζ^σ	σ^E disaster spike	20.00%	GD idiosyncr. volatility spike
$\underline{\kappa}^H$	Kelly's skew. of F_t^H in disasters	-0.21	Guvenen, Ozkan, and Song (2014)
		Firms	
α	CD coefficient on labor	0.667	NIPA Average Labor Share of GDP
δ	Capital depreciation rate	7.81%	BEA Fixed Investment
τ	Corporate profit tax rate	25%	Average post-TCJA tax rate
τ^{int}	Debt tax shield	0.235%	Tax rate × average interest rate
		Household	
σ_H	Std.dev. human capital shocks	9.21%	See text
$\rho_{H,E}$	Corr. human capital & firm shocks	0.125	SCF 2019 (see text)

4.2 Calibrated Parameters

Table 2 lists all parameters that we set externally based on observed values in the data. Detailed discussion of each parameter is presented in Appendix C.1. In this section, we focus on parameters whose values are chosen to match moments produced by the simulated model.

Table 3 lists all targeted moments by category. Each moment is assigned a parameter that is most closely identified by this moment. The table displays the resulting parameter value and the model fit respective of these targeted moments. Unless otherwise noted, all moments are annual and in percent.

To generate the model moments in Table 3, we run 80 × 10,000 quarter simulations (each with 500 period "burn-in") and report bootstrapped statistics. The model-generated values for all but the first two moments are computed from a sample conditional on no disaster realization.

These moments are matched to data counterparts computed in the "disaster-free" post WWII sample. The first two moments, however, which are the volatility of consumption growth and the investment rate, are taken from the longer post-WWI sample in the data. We discretize the disaster intensity process ζ_t as a 3-state Markov chain using the Rouwenhorst (1995) method – this yields three possible disaster sizes, which we call small, medium, and large. To calculate volatility of consumption growth and the investment rate in the model, we condition on a sample with small and medium disasters.[18]

Table 3: Jointly Calibrated Parameters

Moment	**Par.**	**Value**	**Model**	**Data**	**Source**
			Macro		
Annual C gr vol, disasters	μ_ζ	0.318	3.41	3.29	Jorda et al. (2016), 1919-2017
Inv Rate vol	ϕ	10	0.846	0.662	Jorda et al. (2016), 1919-2017
Annual C gr vol, no disasters	γ	3.5	2.09	1.89	Personal Consumption Exp. 1954-2019
			Firms		
Leverage, mean	θ	0.7	32.4	37.0	Elenev et al. (2021)
Net payout rate, mean	ξ_0	0.14	11.09	7.81	Elenev et al. (2021)
Annual Div gr, vol	ξ_1	11	7.1	11.0	Kaltenbrunner and Lochstoer (2010)
			Idiosyncratic Risk Shares		
Portfolio	α_O	0.0409	34.0	35.2	Bach et al. (2020)
Private Assets	$\bar{\sigma}_E$	0.16	76.0	76.1	Bach et al. (2020)
			Asset Prices		
Risk-Free Rate, mean	β	0.76	1.11	1.22	Real 3-month Tbill 1960-2019
Market Excess Return, mean	σ	6.65	6.17	6.40	CRSP Excess Log Return 1990-2019
Private Excess Return, mean	ψ	0.0117	12.8	13.1	Xavier (2021)
Market Excess Return, vol	σ_ζ	0.25	16.2	16.4	CRSP Excess Log Return 1990-2019
Market Return Predictability	ρ_ζ	0.9	0.078	0.082	CRSP Excess Log Return 1926-2019
			Untargeted Moments		
Capital / Ann Output			1.63	2.29	NIPA Fixed Assets 1954-2019
Annual C gr vol / Y gr vol			0.602	0.658	Jorda et al. (2016) 1919-2019
Risk-Free Rate, std			2.55	2.10	Real 3-month Tbill 1960-2019
Net Firm Issuance Rate, mean			2.35	3.08	Elenev et al. (2021)
Dividend Predictability			-0.163	-0.088	CRSP Dividends 1926-2019
Leverage, vol			2.04	3.46	Elenev et al. (2021)

Consumption Volatility and IES. Given these sample definitions, the model matches closely the volatility of consumption growth of 3.29% in the post WWI (1919-2017) sample.

[18]Given the calibrated values for μ_ζ and σ_ζ in Table 3, the medium and large disasters involve TFP drops of 27% and 49%, respectively. The small "disaster" causes a jump in TFP by 3.5% – if we set this value to zero instead, results would be unaffected.

The investment rate in the model is slightly too volatile. As is common, the volatility of investment in the model is closely tied to the magnitude of adjustment costs ϕ. Consumption growth volatility is mainly driven by average disaster size μ_ζ. We choose the inverse of the IES $\gamma = 3.5$ to match consumption growth volatility in the post WWII (1954-2019) sample to the model-generated volatility in the no-disaster simulation. The resulting IES of 1/3 is close to microeconomic estimates and in line with numbers used by many macro papers.

Asset Prices. The model closely matches the targeted asset prices. As usual, the discount factor β_H is determined by matching the average risk-free real rate – an annual discount factor of 0.76 yields an interest rate of 1.11%, indicating a large precautionary motive. To match the aggregate equity risk premium, i.e. the excess return on a fully diversified portfolio, the model needs risk aversion of 6.65. The idiosyncratic risk variance shares for the portfolio held by the representative investor, and the hypothetical fully concentrated portfolio, are computed with data taken from Bach et al. (2020). These targets are laid out in Section 2. They identify two key model parameters, the dispersion of idiosyncratic capital shocks in no-disaster states $\bar{\sigma}_E$ and the factor share of managerial effort in production α_O. Given this overall level of risk premia, the utility benefit from holding a concentrated stake ψ is identified from the slope of the idiosyncratic risk curve. A value of 0.0117 matches closely the value of the excess return on private assets computed by Xavier (2021). Since these parameters are novel and unique to our model, we discuss their identification and the estimated values in Section 5.1.

The time-varying disaster severity goes a long way in helping the model to generate sufficiently volatile returns:[19] the model generates the right amount of equity return volatility of 16.2% even in the no-disaster sample, by creating large time-variation in the stochastic discount factor. This is due to substantial variation in the size of the potential next disaster parameterized by $\sigma_\zeta = 0.25$. The model also generates the right predictability in equity returns, which pins down the persistence of the disaster intensity process ρ_ζ.[20]

[19] Wachter (2013) generates both the level and volatility of the aggregate equity excess return in a model with exogenous consumption. Gourio (2012) matches the data volatility, yet overshoots on the level of the risk premium in a production model. Our model delivers both the numerator and denominator of the Sharpe ratio.

[20] The reported value is the coefficient on the log dividend-price ratio dp_t in the following regression, run using annual observations both in model and data: $r^m_{t+1} = \alpha_r + \beta_r dp_t + \epsilon^r_{t+1}$, where r^m_{t+1} is the realized log return on the market portfolio.

Financial Frictions and the Excess Volatility Puzzle. Financial frictions in the shape of equity issuance costs and the leverage constraint are key features that allow the model to match the high volatility of equity excess returns while simultaneously generating the low volatility of corporate payouts in the data. Firms in our model choose leverage subject to a collateral constraint that conditions on the lowest possible realization of the capital price q next period i.e. $\theta_t = \theta \min_t q_{t+1}$. Disasters cause a large drop in q, restricting actual firm leverage to be significantly lower than maximum leverage θ in a riskless world. With a maximum leverage ratio of 0.7, the model matches average leverage of U.S. non-financial businesses closely. The magnitude of the equity issuance cost ξ_1 targets volatility of dividend growth, and the target payout rate ξ_0 targets the average net payout rate (dividend + repurchases - issuance). Table A.1 in Appendix D.1 highlights the effects of these frictions by comparing the calibrated baseline to two other economies in which financial frictions are tuned off. Absent equity issuance costs, the annual volatility of corporate payouts would be 19% in the model, close to double the data. At the same time, excess return would be too high, yet not volatile enough, leading to a 56% Sharpe ratio, much higher than in the data. In other words, having financial frictions allows our model to overcome the excess volatility puzzle. Given our goal of estimating the relative importance of idiosyncratic risk, a model that misses the volatility of aggregate returns and that overstates the Sharpe ratio of market returns would severely bias our estimates. In particular, such a model would lead to underestimates of the volatility of firm shocks $\bar{\sigma}_E$ and the importance of managerial effort α_O.

4.2.1 Untargeted Moments

Table 3 also reports several untargeted moments. The model generates a capital to output ratio of 1.63 that is close to the data. The ratio of consumption growth to output growth volatility is low in the model but close to the data, as is the volatility of leverage; at the same time, the risk free rate is somewhat too volatile. Both are the result of a low IES that we have fixed externally. Given the financial frictions parameterized by the payout target ξ_0 and the issuance cost ξ_1, firms on average issue 2.35% new equity every year, close to the data value of 3.08%. This implies that on net firms pay out 11.09% of their equity value every quarter, relative to 7.81% in the data. The model produces the right sign for dividend predictability, even though dividends in the model are too negatively correlated.[21]

4.2.2 Asset Price Dynamics

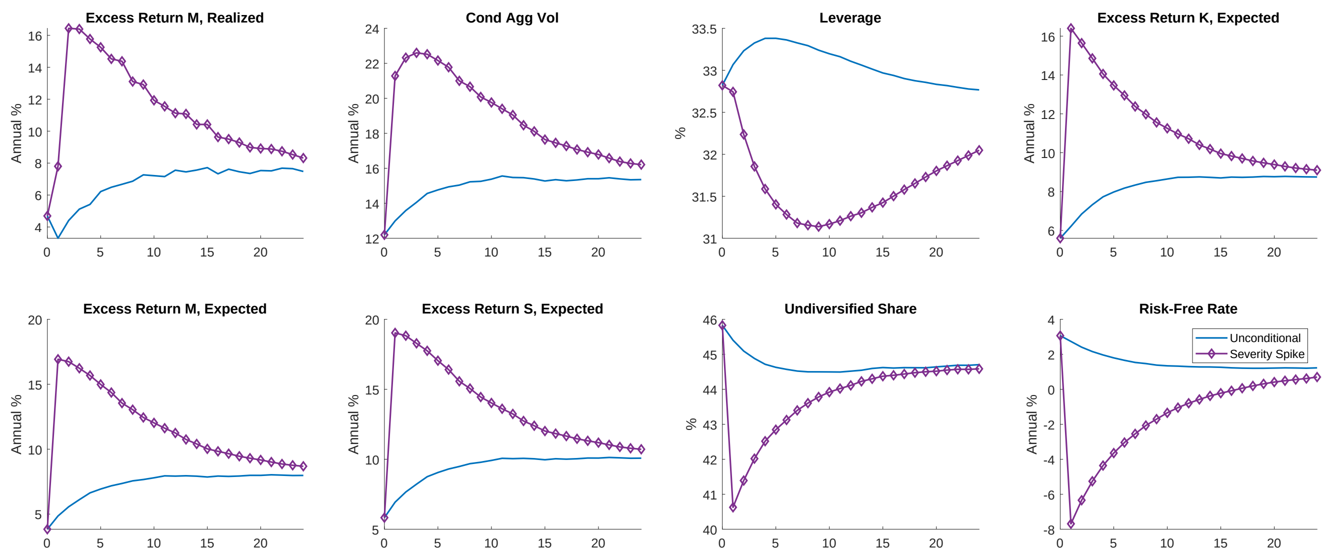


Figure 4: Spike in Disaster Severity: Financial Variables

Shocks to disaster severity lead to large swings in asset prices and expected returns, as displayed in Figure 4.[22] This figure also shows the impulse response to a jump in disaster severity from moderate to large. Since the severity process only has three possible states, the graphs show the unconditional evolution of all variables as the solid blue line. The panels in the bottom row show expected excess returns on physical capital, the diversified market portfolio, and equilibrium portfolio. The market risk premium spikes to 17% initially and slowly declines as disaster severity mean-reverts. Realized excess returns in the top left panel initially drop as the value of firm equity declines, but then follow the pattern of excess returns. Spikes in disaster severity are times of high conditional uncertainty: the middle panel in the top row shows that the conditional volatility of the market excess return rises from 14% to 24%. These impulse responses demonstrate how the model generates realistically large return volatility. Firms respond to the higher disaster risk by reducing leverage, shown in the top right panel.

[21] The reported value is the coefficient on the log dividend-price ratio dp_t in the following regression, run both in model and data: $\Delta d_{t+1} = \alpha_d + \beta_d dp_t + \epsilon^d_{t+1}$, where Δd_{t+1} is the log growth rate in dividends from t to $t+1$.

[22] Appendix D.2 discusses the response of macro quantities to disaster realizations as well as changes in disaster severity.

Large disasters cause Tobin's q to fall by more than moderate disasters, effectively tightening the firm's leverage constraint. To maintain a precautionary buffer, firms cut back on debt.

4.3 Return on Wealth and Wealth Inequality

The wealth distribution in our model does not matter for aggregate asset prices by construction. Nonetheless, the model produces a reasonable amount of wealth concentration. Human capital is subject to undiversifiable idiosyncratic risk, while households optimally choose to retain exposure to firm idiosyncratic risk. These exposures lead to volatile returns on wealth for individual households and imply a great deal of wealth inequality.

The largest position in the household portfolio is human capital, accounting for nearly three-fifths of total wealth. Inability to insure against human capital shocks also makes this the most volatile component. About a fifth of household wealth is invested in equity, with the remainder split about equally between managerial capital and bonds. The total return on wealth is the return on this portfolio. Outside disasters, its annualized volatility is 18%, higher than the the volatility of the market portfolio (16.2%) but notably lower than the volatility of partially diversified equity portfolio actually held by households (20.0%) due to their risk-free bond holdings. The fact that household wealth is no riskier in our model than concentrated equity positions are in the model and data suggests that the successful resolution of asset pricing puzzles in this paper does not rely on counterfactually large volatility of the return on wealth.

Successive realizations of idiosyncratic shocks cause wealths of ex-ante identical households to diverge. This is consistent with Xavier (2021), who argues that the primary driver of wealth inequality in U.S. data is heterogeneous returns on wealth (rather than, e.g., heterogeneous savings rates). To construct the wealth distribution, however, we must modify the model slightly.

In our baseline model, households are infinitely-lived and idiosyncratic shocks are permanent, causing a diverging wealth distribution. To achieve a stationary distribution, we introduce mortality in a "perpetual youth" sense. Every period, households die with probability π. Their bequests are pooled and distributed equally to measure π of newly born households. This is an innocuous modification of the model – it simply requires the discount factor β above to be reinterpreted as a product of the discount factor $\tilde{\beta}$ of the mortal households with their survival

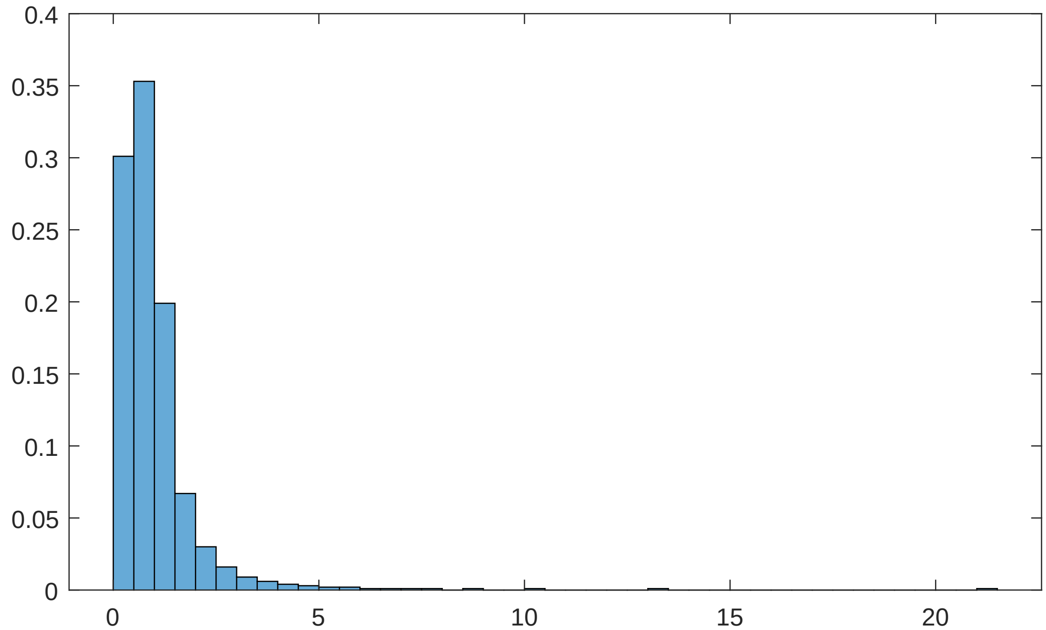


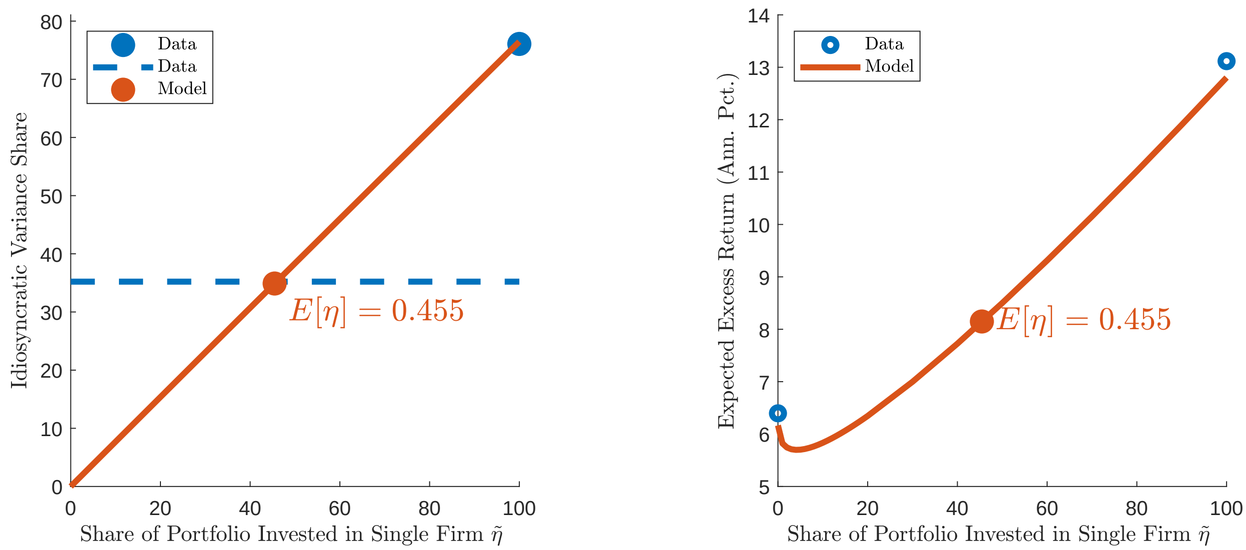
Figure 5: Wealth Distribution As Share of Mean

probability $1 - \pi$. We calibrate $\pi = 0.44\%$ quarterly, consistent with households "born" at age 20 and living on average until 77.3, the average life expectancy in the U.S.[23].

Figure 5 plots the wealth distribution relative to the mean. The poorest percentile of households have less than 3% of the wealth of the average household, while the richest have more than 5.4x the average wealth. Two-thirds of households have wealth below the average, with the richest decile owning a third of all wealth. The model does not produce the full wealth concentration observed in the data. Limiting the 2019 Survey of Consumer Finances sample to positive-net-worth equity owners yields a median that is less than a quarter of the average, and a top decile that owns more than two-thirds of all wealth.

There are two reasons for this discrepancy. First, the SCF does not include human capital in net worth, while the model does. Labor income is less concentrated than wealth – a simple capitalization of SCF wage income using the average growth rate and discount rate from the model lowers the share of wealth held by the top decile by more than 10 percentage points.

[23]More precisely, the EZW preferences in our model imply the transformation $\beta = \tilde{\beta}(1-\pi)^{\frac{1-\gamma}{1-\sigma}}$, see for example Gomes and Michaelides (2007). To compute the distribution, we draw a sample of 10,000 households with idiosyncratic shock realizations for each period of the simulation and cumulate the idiosyncratic returns on wealth to construct a wealth distribution in each period. After discarding the first 1,000 periods to ensure a stable distribution, we average across time to produce the ergodic steady state wealth distribution.



(a) Idiosyncratic Variance Shares

(b) Idiosyncratic Excess Returns

Figure 6: Identification of Idiosyncratic Risk Parameters

Second, the model assumes ex-ante identical households and egalitarian redistribution of bequests, both of which bias the wealth distribution towards less dispersion. Ultimately, a richer model – one that would not admit the kind of analytical aggregation employed in this paper – is needed to shed light on the implications of under-diversification for wealth inequality.

5 Results

5.1 Identification of Idiosyncratic Risk Parameters

In Section 2, we laid out the key empirical targets for model estimation. Our first goal is to infer through the model the magnitude of idiosyncratic risk borne by the marginal investor, as well as the sources that make this level of portfolio concentration the (privately) optimal choice. Figure 6 illustrates how the model matches these facts. The average level of portfolio concentration in the model is 45%. With this level of concentration, the model matches the idiosyncratic risk shares from Bach, Calvet, and Sodini (2020) in the left panel and excess returns on the market portfolio and the fully private portfolio in the right panel. We will discuss the shape of the excess return line on the right side in the next section.

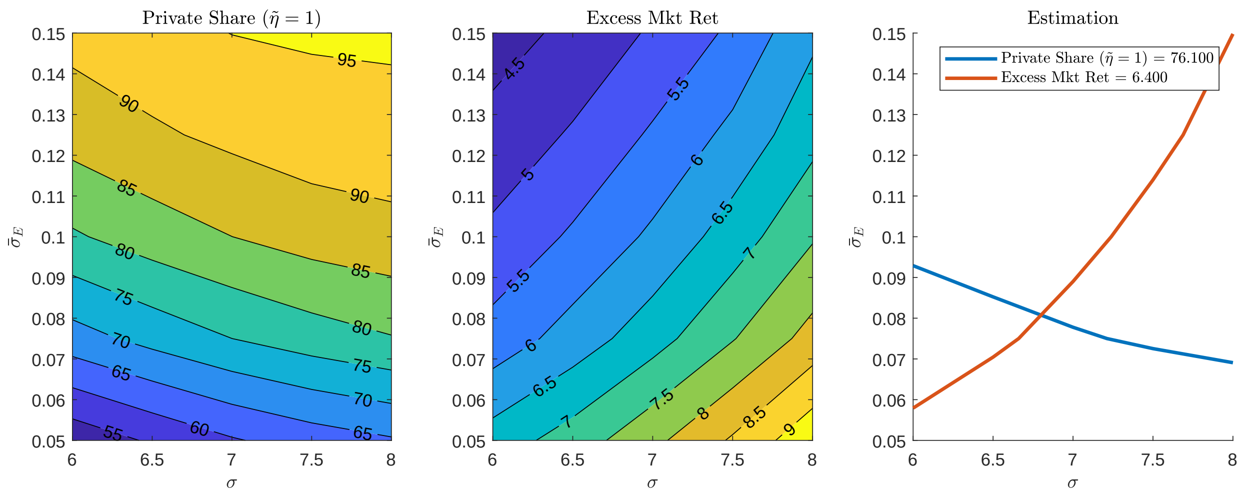


Figure 7: Identification of Concentrated Portfolio Share and Idiosyncratic Risk

Identification of $\bar{\sigma}_E$ and σ. In order to identify the representative investor's effective exposure, the model needs to match the relative contribution of aggregate and idiosyncratic return risk for the fully concentrated portfolio, see panel (a) of Figure 6. At the same time, the model needs to match the market equity premium. Jointly these moments tightly identify two key model parameters: the standard deviation of idiosyncratic shocks $\bar{\sigma}_E$ and risk aversion σ. Identification of these parameters is best understood from plotting their joint effects on two model moments: the idiosyncratic variance share of the fully concentrated ($\tilde{\eta} = 1$) portfolio and the market equity premium.

The left two panels of Figure 7 plot isoquants for these two model moments, respectively, as functions of $\bar{\sigma}_E$ and η, holding fixed all other parameters at their estimated values from Table 3. As expected, increasing the fraction of idiosyncratic firm risk $\bar{\sigma}_E$ raises the idiosyncratic risk share for every level of risk aversion. Similarly, higher risk aversion leads to a higher risk premium. The plots also reveal that everything else equal, more idiosyncratic risk $\bar{\sigma}_E$ leads to a lower aggregate risk premium. This is because greater $\bar{\sigma}_E$ raises the costs of choosing a concentrated portfolio and therefore lowers equilibrium η_t. As a result, the price of aggregate risk declines and with it all risk premia. The pronounced nonlinearities in isoquant lines highlight the need to estimate both parameters jointly in an equilibrium asset pricing model. The estimate is precisely determined at the intersection point of both target isoquants in the right panel of Figure 7.

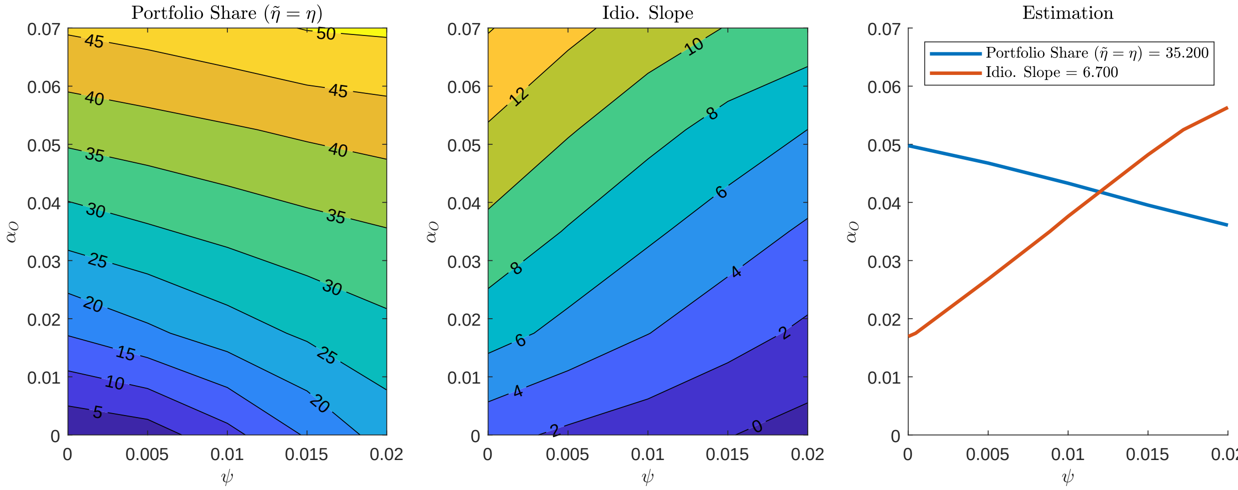


Figure 8: Identification of Risk Aversion and Labor-Equity Correlation

Identification of α_O and ψ. We can apply the same approach to the joint identification of the two key sources of non-diversification in the model: the importance of managerial effort α_O and the utility benefit from owning a business ψ. The two left panels of Figure 8 plot the two target moments: the idiosyncratic risk share of the equilibrium portfolio ($\tilde{\eta} = \eta_t$) and the idiosyncratic risk premium slope, defined as the difference between the excess returns earned on a fully concentrated and fully diversified portfolio. The left panel shows that higher α_O raises the share of idiosyncratic risk in the equilibrium portfolio; it does so by raising the benefits of choosing a more concentrated portfolio (higher η_t) in order to provide more managerial effort. This mechanism is intuitive: when managerial effort is more important for production, households earn higher fees f_t when providing this effort, incentivizing higher concentration. Similar to α_O, a higher utility benefit ψ also raises the idiosyncratic risk in the equilibrium portfolio.

The effect of both parameters on the slope of the idiosyncratic risk line is different, however. Here, higher α_O increases the risk premium investors demand for holding idiosyncratic risk. The utility benefit ψ, on the other hand, lowers the required risk premium since it provides non-pecuniary compensation for taking on this risk. This differential effect of both motives for non-diversification is the key identifying force in the model.

Both parameters are uniquely identified at the intersection point in the right panel.

Are the Estimates Reasonable? Above we argue that our model achieves precise structural identification of the novel parameters $\bar{\sigma}_E$, α_O, and ψ through an intuitive mechanism. But are the resulting parameter estimates reasonable when compared to the data? To gauge the magnitude of quarterly idiosyncratic volatility $\bar{\sigma}_E$ at 8.21%, we can compare this estimate to measures of cross-sectional dispersion in idiosyncratic firm productivity shocks or sales. Bloom et al. (2018) estimate the quarterly unconditional standard deviation of persistent (AC of 0.95) yet transitory shocks to idiosyncratic productivity at 12.5% outside of recessions. Salgado et al. (2020) estimate equally persistent shocks to sales with a higher unconditional standard deviation of 22% in low-risk states of the world. Our shocks are i.i.d., but have *permanent* effects since they scale the capital stock of a firm in addition to affecting the current period's profit. Since our shocks do not mean-revert, the somewhat smaller magnitude of innovations is comparable to the transitory shocks estimated by other papers.

The estimate of the importance of managerial effort $\alpha_O = 0.0409$ implies that compensation of managers should account for roughly 4% of revenues, or equivalently 6.5% of total labor income. We can broadly view this number as an estimate of the productive benefit that can justify the large exposure to idiosyncratic risk seen in the data. To check whether this number is sensible, we compute compute the share of compensation received by managers in two datasets. First, we compute total executive compensation as share of sales for publicly listed companies in the Compustat 2021-2022 sample, which amounts to 2.8%. Second, using the 2022 wave of the Current Population Survey, we compute the income of CEOs and other top managers (occupation codes 10 and 20) as share of the total wage bill, which is 4.5%. Both numbers have the same magnitude and are slightly below our estimate. There are various reasons to expect some difference to our model-implied estimate – in particular, our definition of managers might be too narrow.[24]

Our estimate of the utility benefit $\psi = 0.0117$ is best understood as non-pecuniary compensation for taking on idiosyncratic risk. ψ is the crucial element that allows the model to fit the idiosyncratic risk line in panel (b) of Figure 6. With $\psi = 0$, the model-implied slope of this line would be too steep, with investors requiring a much larger than observed return on undiversified positions. Based on Figure 10, we calculate that our estimate of ψ is equivalent to a 3pp lower return on a fully concentrated position. Put differently, households in our model

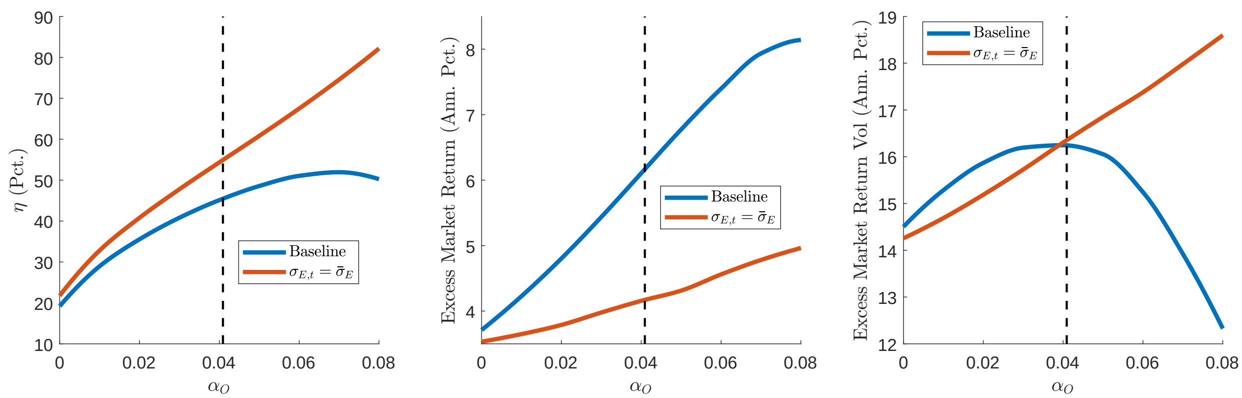
[24]If we include compensation of all "manager" occupation codes in the CPS calculation, the share is close to 20%, but is probably subject to considerable manager title inflation.

enjoy "being their own boss" to the extent that they are willing to give up 3pp equity return in exchange. This preference for control contributes to our model's resolution of the private equity premium puzzle.

Finally, we note that the implied level of non-diversification $\mathrm{E}[\eta] = 44.7\%$ is very close to the simple calculation we performed in Section 2, which yielded a "naive" estimate of 46%.

5.2 Is Imperfect Diversification the Answer to Asset Pricing Puzzles?

A key question of our analysis is to which extent quantitatively realistic exposure of the representative investor to idiosyncratic risk can help explain asset prices, in particular the magnitude and volatility of excess returns on equity.



(a) Implied $\mathrm{E}[\eta_t]$ (b) Level (c) Volatility

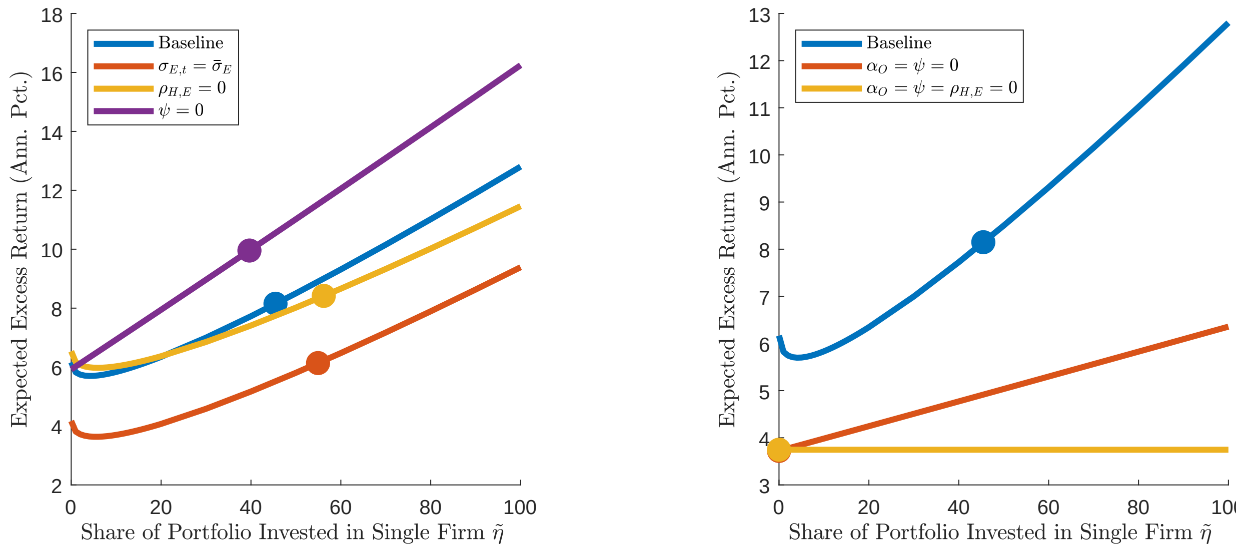
Figure 9: Idiosyncratic Exposure and Aggregate Risk

Aggregate Equity Premium. Figure 9 provides the answer to this question in the context of our estimated model. It shows the risk premium on the fully diversified market portfolio (panel (b)) and its volatility (panel (c)) for different levels of α_O. The blue "Baseline" plots risk premiums with all parameters held fixed at their estimated values from Table 3 and only varying α_O. In panel (a), we can see how the level of concentration chosen by investors varies with α_O – generally, greater α_O leads greater η_t, but at high levels of α_O the slope starts to reverse. The dashed vertical line marks the estimate of $\alpha_O = 0.0409$, at which point the model generates the data equity premium of 6.2% in panel (a) and its volatility in panel (b). If we reduced α_O counterfactually to zero, the risk premium would drop to 3.7%, implying that lack of diversification explains roughly 40% of the aggregate equity premium. Conversely, if α_O was doubled to 0.08, the market equity premium would be 9%. With respect to volatility, if we reduced α_O to zero, it would fall from 16.3% to 14.5%, thus explaining roughly 15% of observed aggregate market volatility.

The graph also quantifies the main force responsible for this large contribution to aggregate risk prices. The red line turns off time-variation in the standard deviation of idiosyncratic firm shocks. In this version of the model, $\sigma_{E,t}$ is constant and thus does not rise during disasters. As the red line in panel (a) shows, in this case households increase η more aggressively with higher α_O, as the cost of non-diversification is much smaller. Despite the much larger rise in η, the market risk premium in panel (b) barely rises in α_O. This result confirms findings in the prior literature, in particular Krueger and Lustig (2010), that exposure to state-independent idiosyncratic risk does not increase aggregate excess returns. However, our simple empirical analysis in Section 2 suggests that time-variation in idiosyncratic risk is likely substantial.

The pattern of variation in excess return volatility in panel (b) is highly non-monotonic. Volatility peaks around α_O of 0.04 and drops to 12.5% as α_O increase to 0.08. This non-monotonicity is the result of opposing effects: higher α_O initially raises the stochastic discount factor's exposure to variation in $\sigma_{E,t}$ through higher optimal η, which ceteris paribus increases excess return volatility. However, this greater exposure also causes firms to make more conservative payout and leverage choices, which counteract the rise in SDF volatility.

Idiosyncratic Equity Premium. Figure 10 considers the effect of different model forces on the price of idiosyncratic risk. The graph shows excess returns on portfolios with different degrees of diversification, holding constant the equilibrium diversification of the representative investor. Panel (a) fixes α_O at the estimated value of 0.0409. The blue line plots again the baseline version of the model that matches the data line from Figure 6 exactly at both ends. By construction, the intercept of the blue line in panel (a) of Figure 10 is equal to the intersection of the blue line with the vertical dashed line in panel (a) of Figure 9 – both represent the market equity premium in the baseline model. The curve is not straight because of the decreasing marginal non-pecuniary benefit of concentration. When $\tilde{\eta}$ is low, the household is willing to pay a lot to increase concentration – the higher utility benefit of concentration offsets the



(a) Baseline $\alpha_O = 0.0409$

(b) Full Diversification

Figure 10: Marginal Price of Idiosyncratic Risk

higher cost of idiosyncratic risk exposure and expected return decreases. But as concentration $\tilde{\eta}$ grows, expected returns load more on idiosyncratic risk, producing an increase and roughly linear relationship between $\tilde{\eta}$ and expected returns.

When we turn off time-variation in $\sigma_{E,t}$ (in red), the line shifts downward but does not change its slope. As we keep $\sigma_{E,t}$ constant, the market risk premium given by the intercept declines to 4%. Since the slope is unaffected, we conclude that time-variation in the quantity of idiosyncratic risk has large effects on the overall level of risk premia, but little effect on the marginal price of idiosyncratic risk. On the contrary, the correlation ρ_{HE} has a pronounced effect on the slope. With ρ_{HE} counterfactually set to zero, the model cannot match the increase in idiosyncratic excess returns. These counterfactuals reveal that the high returns on undiversified business stakes we observe in the data are partly due to correlated labor income risk. If investors could untangle their labor income entirely from their investments, the required yield on fully concentrated positions ($\tilde{\eta} = 1$) would be 1.6% points lower than in the data. The utility benefit ψ has an even larger effect on the slope. With ψ counterfactually set to 0, the slope would be a lot larger, as illustrated by the purple line. The fully concentrated portfolio return would be 16% instead of 12.8% in the baseline. The estimated benefit of $\psi = 0.0117$ is therefore equivalent to having 3.2pp higher return on a fully undiversified portfolio.

Panel (b) of Figure 10 considers a different counterfactual: it compares the baseline marginal price of idiosyncratic risk to a world in which full diversification is optimal, with $\alpha_O = \psi = 0$. As one would expect, the intercept shifts down below the 4% counterfactual risk premium from panel (a) of Figure 9. However, even when $\alpha_O = \psi = 0$, the slope of the risk line does not go all the way to zero. This is a subtle prediction of our model: even investors who do not hold any undiversified business equity demand some compensation for a marginal increase in idiosyncratic risk exposure. Comparison to the yellow line in panel (b) reveals the reason for this effect. Once we set $\rho_{HE} = 0$ in addition to full diversification, the slope of the line becomes zero. However, as long as $\rho_{HE} > 0$, even fully diversified investors retain some exposure to idiosyncratic firm risk in their stochastic discount factor because this firm risk is correlated with their individual labor income risk. Therefore, if we were to use the SDF of a diversified investor to "price" a cash flow with exposure to the idiosyncratic risk of their employer, this risk would be priced.

5.3 The Real Effects of Idiosyncratic Risk

The previous section demonstrated that taking into account investor exposure to idiosyncratic risk has quantitatively large effects on asset prices. We can take this analysis a step further and use our production model with endogenous output, investment, and consumption to study the real effects of concentrated portfolios. Table 4 compares the baseline to several counterfactual economies.

Constant Idiosyncratic Risk. The first two comparison columns hold fixed the idiosyncratic risk of firm shocks ($\sigma_{E,t} = \bar{\sigma}_E$), column (1), and of firms and human capital ($\sigma_{E,t} = \bar{\sigma}_E$ and $\sigma_{H,t} = \bar{\sigma}_H$), column (2). Eliminating time-variation in idiosyncratic risk reduces the cost of choosing a high η_t – average η therefore increases. Even though equilibrium η is higher, expected excess returns on all assets are lower and the risk free rate is higher. This reflects a decline in the compensation for aggregate risk as well as idiosyncratic risk. When investors are exposed to risk, their precautionary savings demand is greatly reduced. While risk free rates increase, the required excess return on physical capital declines. However, in equilibrium the risk free rate effect dominates, with the total discount rate for capital increasing to 10.36% from 8.88%. As a result, investment, the aggregate capital stock and output are lower in the economies with constant risk. In the economy that keeps both sources of risk constant, con-

Table 4: Effects of Concentrated Portfolios

Moment	**Base**	$\bar{\sigma}_E$ **(1)**	$\bar{\sigma}_E, \bar{\sigma}_H$ **(2)**	α_O **(3)**	ψ **(4)**	**FD (5)**	τ^η **(6)**
	Idiosyncratic Risk						
Concentration η_t	45.5	54.9	58.6	19.2	39.7	0.0	47.9
Risk Share, Portfolio ($\tilde{\eta} = \eta_t$)	34.0	41.5	38.0	10.2	29.8	0.0	38.8
Risk Share, Private ($\tilde{\eta} = 1$)	76.0	70.2	63.9	75.3	76.8	75.4	78.2
	Returns						
Risk-Free Rate	1.11	3.08	3.62	5.07	3.82	7.12	0.85
Market E ER	6.17	4.13	3.83	3.68	5.89	3.70	6.30
Portfolio E ER	8.28	6.21	5.95	2.83	10.06	3.70	9.01
Private E ER	12.80	9.34	8.53	4.29	16.22	3.70	13.86
Human Capital E ER	20.7	18.5	17.5	19.2	20.8	19.3	20.7
	Return Volatility						
Risk-Free Rate, vol	2.55	2.05	1.92	1.78	2.59	1.89	2.64
Market R ER, vol	16.2	16.3	18.1	14.5	16.0	14.3	15.6
Portfolio R ER, vol	20.0	21.3	23.0	15.3	19.1	14.4	19.9
Private R ER, vol	33.2	30.0	30.2	29.2	33.3	29.0	33.4
	Firms						
Leverage	32.42	31.80	28.57	32.43	32.95	33.30	33.21
Dividend gr, vol	7.11	6.85	6.54	9.48	6.76	8.72	7.07
Phys. Capital E ER	7.77	6.61	6.74	3.57	6.60	2.76	8.15
Phys. Capital Total Ret	8.88	9.69	10.36	8.64	10.42	9.88	9.00
Net Payout Rate	11.09	11.35	11.70	8.51	12.58	9.68	11.58
constraint binds (%)	13.40	0.00	0.00	0.00	0.00	14.81	0.00
	Production (unconditional pct change rel. to Base)						
Output	1.03	-1.24	-3.02	6.00	-5.35	1.99	0.11
Consumption	0.87	-0.32	-1.38	5.38	-3.73	2.97	0.22
Capital / Output	1.64	-6.20	-11.90	7.74	-13.81	-4.04	-0.31
Consumption gr, vol	4.58	-17.56	-21.35	-22.93	5.64	-15.58	4.12
	Welfare (unconditional pct change rel. to Base)						
Cost of η_t	4.58	2.96	2.99	-78.86	-24.71	-100.00	10.05
HH Wealth / Output	23.33	-1.27	-3.20	-7.54	-17.15	-18.15	2.27
Welfare / HH Wealth (v^H)		1.46	2.87	15.13	15.57	27.11	-2.20
Welfare		0.17	-0.46	6.52	-4.21	4.15	0.06

sumption is 1.38% lower and consumption growth is 21.35% less volatile. These two effects almost offset each other in terms of the net effect on aggregate (utilitarian) welfare, with the constant $\bar{\sigma}$ economy in (2) having 0.46% lower welfare compared to the baseline.

Eliminating Sources of Non-Diversification. Columns (3) to (5) move from the baseline economy to a standard production economy with rare disasters by eliminating the different source of portfolio concentration. Column (3) sets $\alpha_O = 0$, thereby shifting the weight of this parameter in the production function from managerial effort to regular labor. In this economy, the only benefit from choosing $\eta_t > 0$ is due to utility ψ, and consequently equilibrium η is lower at 19.2%. The reduction in risk exposure causes a large increase in the risk free rate and significantly smaller risk premiums on all risky assets. At the same time, greater labor supply raises output and consumption. The net effect is a larger economy with less risk. Column (4) keeps α_O fixed at the baseline estimate, but instead sets $\psi = 0$. In this economy, the sole reason for choosing $\eta_t > 0$ is to earn income on managerial effort. The result is again slightly lower equilibrium η_t and lower excess returns on the market and physical capital. Required returns for idiosyncratic firm risk, however, are much higher. The $\psi = 0$ economy suffers from an "undersupply" of managerial effort relative to the baseline, yielding a substantially smaller economy. Column (5) combines (3) and (4), and in addition also sets the correlation $\rho_{H,E} = 0$. Economy (5) features full diversification ("FD") and is a standard neoclassical production economy with rare disasters. Holding the other parameters fixed, we can see that this economy misses the data in many dimensions – the risk free rate is 7.12% as opposed to 1.11% in the baseline. The market risk premium is 3.70% and the excess return on physical capital is 2.76%. The economy features a 4.04% smaller capital stock, yet 2.0% higher output. Households are less wealthy, but aggregate welfare is 4.15% higher.[25] Comparing economies (3) to (5) to the baseline economy shows that accounting for lack of diversification and its economic sources fundamentally changes the behavior of the standard asset pricing model.

Quantifying the Welfare Cost of Undiversified Exposure. The counterfactuals in Table 4 provide a decomposition of our model's departures from a typical production-based asset pricing model. For each of columns of the table, we also compute the direct welfare cost of exposure to idiosyncratic firm risk in the row "Cost of η_t." We obtain this number by calculating an alternative household value function $v^{H,0}(\mathcal{Z}_t)$ that assumes $\eta_t = 0$.[26] This value function is

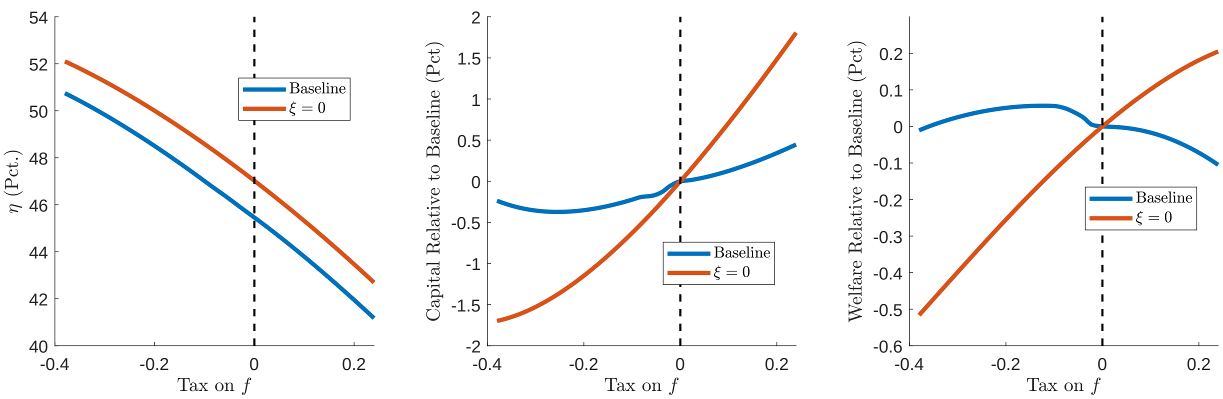
[25] Since we are comparing economies with different preferences and technologies, welfare comparisons do not indicate feasible improvements that could be implemented through policies or by a social planner. Rather, they just highlight the social costs of the technological constraints that necessitate non-diversification.

[26] Technically, we compute this value function based on an alternative certainty equivalent that includes wealth growth under the counterfactual assumption that $\eta_t = 0$.

evaluated at the actual equilibrium policies and prices, i.e. it is not the outcome of households' actual optimization. In the table we report $v^{H,0}(\mathcal{Z}_t)/v^{H,0}(\mathcal{Z}_t) - 1$, which is the hypothetical welfare gain in consumption units from erasing idiosyncratic firm risk in the household portfolio, while holding all choices and prices constant. For the baseline economy, households would be 4.58% better off without this risk. Eliminating the different sources of non-diversification in columns (3) – (5) obviously reduces this cost; in the full diversification economy (5) it is reduced to zero as $\eta_t = 0$ is the optimal household choice.

5.4 Socially Optimal Diversification

Households choose η_t based on first-order condition (29), trading off the private costs and benefits of diversification. However, our model features an externality: households do not internalize the effect of their choice of η_t on the firm SDF and therefore the capital and leverage choice of firms. To highlight this externality, we introduce a tax on managerial fee income. With this tax, the income term on the left of the budget constraint (16) becomes $(1 - \tau^{\eta}) f_t \eta_{t,i} H^O_{t+1,i}$, and the tax introduces a wedge in the FOC for η_t.[27]



(a) Effect on η_t (b) Effect on Capital (c) Net Welfare Gain

Figure 11: Taxing Underdiversification

Panel (a) of Figure 11 shows for the baseline economy in blue, and for a counterfactual economy without equity issuance cost ($\xi = 0$) in red, that the tax works as intended: a higher tax reduces the benefit of supplying managerial effort, causing lower equilibrium η_t. In Panel

[27] Tax revenue is rebated with labor income in the same way as corporate tax revenue. Since labor is supplied inelastically, this does not causes distortions. Note that we cannot implement a lump-sum rebate without breaking our aggregation result for households.

(c), we can see that for the baseline economy, welfare is maximized at a tax $\tau^\eta = -0.16$, a 16% subsidy. In this optimal-tax economy, which is also listed in column (6) of Table 4, average η_t is 47.9%, over 2% higher than in the baseline. The resulting economy has greater idiosyncratic and aggregate risk, a lower risk free rate, higher risk premiums, a smaller capital stock, and slightly greater output. Aggregate welfare is 0.06% higher.

Why is higher concentration socially optimal? The reason for this result is best understood by studying the effect of the same tax in a counterfactual economy without equity issuance costs (red lines). When households choose lower η_t, firms' SDF has lower exposure to capital quality shocks. As a result, firms choose to purchase more capital and increase their size. Scaling up of course also requires more firm equity, which is costless to support when $\xi = 0$. Panel (b) illustrates this effect clearly – a higher tax on f_t leads to a greater equilibrium capital stock, with an increase of about 2% at a 20% tax. In terms of welfare, lower η_t causes lower supply of managerial effort, yet this effect is dominated by the rise in capital and the reduction in risk exposure, making a large positive tax optimal in the model with $\xi = 0$ as shown in panel (c).

However, increasing the scale of the firm becomes costly with equity issuance costs in the baseline, since it requires more equity inside the firm. In the presence of financial frictions, firms therefore grow much less as idiosyncratic risk is reduced through the tax on f_t, as can be seen by comparing baseline to $\xi = 0$ in panel (b). In the baseline economy, it is hence optimal to shrink the size of firms slightly through a subsidy on f_t, saving equity issuance costs. At the optimum, benefits from lower issuance costs and higher supply of managerial effort just dominate the costs of increased risk exposure and less capital.

6 Conclusion

This paper studies the implications of undiversified portfolio holdings on asset prices and real outcomes. We separately identify two forces that give rise to lack of diversification: the need to incentivize managers through undiversified positions, and private benefits of control derived by business owners. We find that both sources of non-diversification are quantitatively important in order to explain idiosyncratic risk in investor positions as well as the returns investors earn on these holdings.

In the estimated economy with lack of diversification that matches the data, risk free rates are lower and risk premiums higher than in a counterfactual full-diversification economy. Idiosyncratic risk exposure can explain a large share – 40% – of the market equity premium in our model. Put differently, our model can explain aggregate asset prices with lower risk aversion, smaller disasters, and a much lower IES than standard asset pricing models.

Finally, we demonstrate that even in our relatively simple model with ex-ante identical households, the welfare implications of non-diversification are subtle, since household choices of optimal portfolio concentration impact firm choices on investment and capital structure. Our estimated model features slightly too much diversification, yet welfare gains from incentivizing more concentrated portfolios are small. Our analysis highlights a complex interaction of idiosyncratic portfolio risk with financial frictions that should be explored in future research.

Our modeling framework also makes a technical contribution. Despite two sources of uninsurable risk, the model admits aggregation and is computationally tractable enough to facilitate estimation. The framework proposed in the paper is well-suited to study aggregate implications of idiosyncratic risk exposures in other contexts. Research into the individual effects of under-diversification would require relaxing the assumptions that make aggregation possible. A richer model in which the distributions of financial and human capital wealth are state variables could be used to study the effect of under-diversification on outcomes such as wealth inequality.

References

Hengjie Ai and Anmol Bhandari. Asset pricing with endogenously uninsurable tail risk. *Econometrica*, 89(3):1471–1505, 2021.

Hengjie Ai, Dana Kiku, and Rui Li. A Quantitative Model of Dynamic Moral Hazard. *The Review of Financial Studies*, 08 2022. hhac059.

Henjie Ai, Dana Kiku, Rui Li, and Jincheng Tong. A unified model of firm dynamics with limited commitment and assortative matching. *The Journal of Finance*, 76(1):317–356, 2021.

Laurent Bach, Laurent E. Calvet, and Paolo Sodini. Rich pickings? risk, return, and skill in household wealth. *American Economic Review*, 110(9):2703–47, September 2020.

Robert J. Barro. Rare disasters, asset prices, and welfare costs. *American Economic Review*, 99(1): 243–64, March 2009.

Harjoat S. Bhamra and Raman Uppal. Does household finance matter? small financial errors with large social costs. *American Economic Review*, 109(3):1116–54, March 2019.

Nicholas Bloom, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry. Really uncertain business cycles. *Econometrica*, 86(3):1031–1065, 2018.

Corina Boar, Denis Gorea, and Virgiliu Midrigan. Why are returns to private business wealth so dispersed? Working Paper 29705, National Bureau of Economic Research, January 2022.

Sylvain Catherine. Countercyclical Labor Income Risk and Portfolio Choices over the Life Cycle. *The Review of Financial Studies*, 35(9):4016–4054, 12 2021.

Hui Chen, Jianjun Miao, and Neng Wang. Entrepreneurial Finance and Nondiversifiable Risk. *The Review of Financial Studies*, 23(12):4348–4388, 11 2010.

Zhiyao Chen and Ilya A Strebulaev. Macroeconomic Risk and Idiosyncratic Risk-taking. *The Review of Financial Studies*, 32(3):1148–1187, 06 2018.

George M. Constantinides and Darrell Duffie. Asset pricing with heterogeneous consumers. *Journal of Political Economy*, 104(2):219–240, 1996.

George M. Constantinides and Anisha Ghosh. Asset pricing with countercyclical household consumption risk. *The Journal of Finance*, 72(1):415–460, 2017.

Massimo Dello Preite, Raman Uppal, Paolo Zaffaroni, and Irina Zviadadze. What is missing in asset-pricing factor models? *SSRN Working Paper*, 2023.

William Diamond and Tim Landvoigt. Credit cycles with market-based household leverage. *Journal of Financial Economics*, 146(2):726–753, 2022.

Alex Edmans and Xavier Gabaix. Executive compensation: A modern primer. *Journal of Economic Literature*, 54(4):1232–87, December 2016.

Alex Edmans, Xavier Gabaix, and Augustin Landier. A Multiplicative Model of Optimal CEO Incentives in Market Equilibrium. *The Review of Financial Studies*, 22(12):4881–4917, 12 2008.

Andrea L. Eisfeldt, Antonio Falato, and Mindy Z. Xiaolan. *Human Capitalists.* NBER Macroeconomics Annual 2022, volume 37, March 2022.

Vadim Elenev, Tim Landvoigt, and Stijn Van Nieuwerburgh. A Macroeconomic Model With Financially Constrained Producers and Intermediaries. *Econometrica*, 89(3):1361–1418, 2021.

Andreas Fagereng, Martin Blomhoff Holm, Benjamin Moll, and Gisle Natvik. Saving behavior across the wealth distribution: The importance of capital gains. Working Paper 26588, National Bureau of Economic Research, December 2019.

Andreas Fagereng, Luigi Guiso, Davide Malacrino, and Luigi Pistaferri. Heterogeneity and persistence in returns to wealth. *Econometrica*, 88(1):115–170, 2020.

Jack Favilukis. Inequality, stock market participation, and the equity premium. *Journal of Financial Economics*, 107(3):740–759, 2013.

Jack Favilukis, Xiaoji Lin, and Xiaofei Zhao. The elephant in the room: The impact of labor obligations on credit markets. *American Economic Review*, 110(6):1673–1712, June 2020.

Ernst Fehr, Holger Herz, and Tom Wilkening. The lure of authority: Motivation and incentive effects of power. *The American Economic Review*, 103(4):1325–1359, 2013.

Xavier Gabaix. Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance *. *The Quarterly Journal of Economics*, 127(2):645–700, 03 2012.

Francisco Gomes and Alexander Michaelides. Asset Pricing with Limited Risk Sharing and Heterogeneous Agents. *The Review of Financial Studies*, 21(1):415–448, 11 2007.

Joao F. Gomes and Lukas Schmid. Equilibrium asset pricing with leverage and default. *The Journal of Finance*, 76(2):977–1018, 2021.

François Gourio. Disaster risk and business cycles. *American Economic Review*, 102(6):2734–66, May 2012.

François Gourio. Credit risk and disaster risk. *American Economic Journal: Macroeconomics*, 5(3): 1–34, July 2013.

Daniel L Greenwald, Matteo Leombroni, Hanno Lustig, and Stijn Van Nieuwerburgh. Financial and total wealth inequality with declining interest rates. Working Paper 28613, National Bureau of Economic Research, March 2021.

Fatih Guvenen, Serdar Ozkan, and Jae Song. The nature of countercyclical income risk. *Journal of Political Economy*, 122(3):621–660, 2014.

John Heaton and Deborah Lucas. Portfolio choice and asset prices: The importance of entrepreneurial risk. *The Journal of Finance*, 55(3):1163–1198, 2000.

Joachim Hubmer, Per Krusell, and Anthony A. Smith. Sources of us wealth inequality: Past, present, and future. *NBER Macroeconomics Annual*, 35:391–455, 2021.

Erik Hurst and Benjamin Wild Pugsley. What Do Small Businesses Do? *Brookings Papers on Economic Activity*, 42(2 (Fall)):73–142, 2011.

Erik G Hurst and Benjamin W Pugsley. Wealth, tastes, and entrepreneurial choice. Working Paper 21644, National Bureau of Economic Research, October 2015.

Felipe S. Iachan, Dejanir Silva, and Chao Zi. Under-diversification and idiosyncratic risk externalities. *Journal of Financial Economics*, 143(3):1227–1250, 2022.

Li Jin. Ceo compensation, diversification, and incentives. *Journal of Financial Economics*, 66(1): 29–63, 2002.

Oscar Jorda, Moritz Schularick, and Alan Taylor. Macrofinancial history and the new business cycle facts. In *NBER Macroeconomics Annual 2016, Volume 31*, pages 213–263. University of Chicago Press, May 2016.

Kenneth L Judd. *Numerical methods in economics*. 1998.

Georg Kaltenbrunner and Lars A. Lochstoer. Long-Run Risk through Consumption Smoothing. *The Review of Financial Studies*, 23(8):3190–3224, 05 2010.

Katya Kartashova. Private equity premium puzzle revisited. *American Economic Review*, 104(10): 3297–3334, October 2014.

Matthias Kehrig. The Cyclical Nature of the Productivity Distribution. *Working Paper*, 2015.

Dirk Krueger and Hanno Lustig. When is market incompleteness irrelevant for the price of aggregate risk (and when is it not)? *Journal of Economic Theory*, 145(1):1–41, 2010.

Haim Levy. Equilibrium in an imperfect market: A constraint on the number of securities in the portfolio. *The American Economic Review*, 68(4):643–658, 1978.

Robert E. Lucas. On the size distribution of business firms. *The Bell Journal of Economics*, 9(2): 508–523, 1978.

Robert C. Merton. A simple model of capital market equilibrium with incomplete information. *The Journal of Finance*, 42(3):483–510, 1987.

Randall Morck, Andrei Shleifer, and Robert W. Vishny. Management ownership and market valuation: An empirical analysis. *Journal of Financial Economics*, 20:293–315, 1988. The Distribution of Power Among Corporate Managers, Shareholders, and Directors.

Tobias J. Moskowitz and Annette Vissing-Jørgensen. The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle? *American Economic Review*, 92(4):745–778, September 2002.

Emi Nakamura, Jón Steinsson, Robert Barro, and José Ursúa. Crises and recoveries in an empirical model of consumption disasters. *American Economic Journal: Macroeconomics*, 5(3):35–74, July 2013.

Eli Ofek and David Yermack. Taking stock: Equity-based compensation and the evolution of managerial ownership. *The Journal of Finance*, 55(3):1367–1384, 2000.

Nikolai Roussanov. Diversification and its discontents: Idiosyncratic and entrepreneurial risk in the quest for social status. *The Journal of Finance*, 65(5):1755–1788, 2010.

Geert Rouwenhorst. Asset pricing implications of equilibrium business cycle models. In Cooley, editor, *Frontiers of Business Cycle Research*. Princeton University Press, 1995.

Emmanuel Saez and Gabriel Zucman. Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Data. *The Quarterly Journal of Economics*, 131(2):519–578, 02 2016.

Sergio Salgado, Fatih Guvenen, and Nicholas Bloom. Skewed Business Cycles. *Working Paper*, 2020.

Lawrence Schmidt. Climbing and Falling Off the Ladder: Asset Pricing Implications of Labor Market Event Risk. *Available at SSRN 2471342*, 2015.

David Schreindorfer. Macroeconomic tail risks and asset prices. *The Review of Financial Studies*, 33 (8):3541–3582, 2020.

Vladimir Smirnyagin and Aleh Tsyvinski. Macroeconomic and asset pricing effects of supply chain disasters. Working Paper 30503, National Bureau of Economic Research, September 2022.

Matthew Smith, Danny Yagan, Owen Zidar, and Eric Zwick. Capitalists in the Twenty-First Century. *The Quarterly Journal of Economics*, 134(4):1675–1745, 07 2019.

Matthew Smith, Owen Zidar, and Eric Zwick. Top Wealth in America: New Estimates under Heterogeneous Returns. *The Quarterly Journal of Economics*, 08 2022. qjac033.

Kjetil Storesletten, Christopher I. Telmer, and Amir Yaron. Asset pricing with idiosyncratic risk and overlapping generations. *Review of Economic Dynamics*, 10(4):519–548, 2007.

Raman Uppal and Tan Wang. Model misspecification and underdiversification. *The Journal of Finance*, 58(6):2465–2486, 2003.

Jessica A. Wachter. Can time-varying risk of rare disasters explain aggregate stock market volatility? *The Journal of Finance*, 68(3):987–1035, 2013.

Ines Xavier. Wealth Inequality in the US: the Role of Heterogeneous Returns. *Available at SSRN 3915439*, 2021.

A Model Appendix

A.1 Solution to Firm Problem

The optimal labor demand and investment choices are characterized by familiar intra-temporal first-order conditions

$$l_t = \left(\frac{\alpha_L A_t o_t^{\alpha_O}}{\hat{w}_t} \right)^{\frac{1}{1-\alpha_L}}, \tag{32}$$

$$o_t = \left(\frac{\alpha_O A_t l_t^{\alpha_L}}{\hat{f}_t} \right)^{\frac{1}{1-\alpha_O}}, \tag{33}$$

$$i_t = \tilde{\delta} + \frac{q_t - 1}{\phi}, \tag{34}$$

where, as defined in the main text, $\hat{w}_t = w_t / Z_t$ and $\tilde{\delta} = \delta + \exp(\bar{g}) - 1$.

Using these solutions, we can express profit per unit of capital r_t^K solely as function of prices and parameters.

To solve the firm problem, we make use of the fact that the objective is homogeneous of degree one in net worth. We define the ratios $k_{t+1} = K_{t+1,i}/N_{t,i}$, $x_t = X_{t,i}/N_{t,i}$, and $b_{t+1} = B_{t+1,i}/N_{t,i}$, omitting i subscripts on the ratio variables which will be identical for all firms. Using these normalized decision variables, we define a value function $v^F(\mathcal{Z}_t) = V^F(N_{t,i}, \mathcal{Z}_t)/N_{t,i}$

$$v^F(\mathcal{Z}_t) = \max_{k_{t+1}, b_{t+1}, x_t} \xi_0 - x_t - \frac{\xi_1}{2} x_t^2 + \mathrm{E}_t \left[\mathcal{M}_{t,t+1}^i n_{t+1,i} v^F(\mathcal{Z}_{t+1}) \right] \tag{35}$$

subject to the budget constraint

$$1 - \xi_0 + x_t \geq k_{t+1} q_t - p_t b_{t+1}, \tag{36}$$

the borrowing constraint

$$b_{t+1} \leq \theta q_t k_{t+1}, \tag{37}$$

and net worth growth

$$\begin{aligned} n_{t+1,i} = \frac{N_{t+1,i}}{N_{t,i}} &= k_{t+1} \epsilon_{t+1,i}^F \left((1-\tau) r_{t+1}^K + (1 - (1-\tau)\delta) q_{t+1} \right) - (1 - \tau^{int}) b_{t+1} \\ &= k_{t+1} \epsilon_{t+1,i}^F R_{t+1}^K - (1 - \tau^{int}) b_{t+1}. \end{aligned} \tag{38}$$

The FOCs for this problem are, attaching multiplier μ_t to the budget and λ_t to the borrowing constraint,

$$(k_{t+1} :)\ 0 = \mathrm{E}_t \left[\mathcal{M}_{t,t+1}^i v^F(\mathcal{Z}_{t+1}) \epsilon_{t+1,i}^F R_{t+1}^K \right] - \mu_t q_t + \lambda_t \theta q_t, \tag{39}$$

$$(b_{t+1} :)\ 0 = \mu_t p_t - \lambda_t - (1 - \tau^{int}) \mathrm{E}_t \left[\mathcal{M}_{t,t+1}^i v^F(\mathcal{Z}_{t+1}) \right], \tag{40}$$

$$(x_t :)\ 0 = -1 - \xi_1 x_t + \mu_t. \tag{41}$$

The envelope condition is

$$v^F(\mathcal{Z}_t) = \xi_0 + \frac{\xi_1}{2}x_t^2 + \mu_t\,(1-\xi_0)\,. \tag{42}$$

We solve the FOC for issuance for the budget multiplier

$$\mu_t = 1 + \xi_1 x_t.$$

The firm's Euler equation for bonds is

$$p_t = (1-\tau^{int})\mathrm{E}_t\left[\mathcal{M}^i_{t,t+1}\frac{v^F(\mathcal{Z}_{t+1})}{\mu_t}\right] + \frac{\lambda_t}{\mu_t}, \tag{43}$$

and the Euler equation for capital is

$$q_t = \mathrm{E}_t\left[\mathcal{M}^i_{t,t+1}\frac{v^F(\mathcal{Z}_{t+1})}{\mu_t}\epsilon^F_{t+1,i}R^K_{t+1}\right] + \frac{\lambda_t}{\mu_t}\theta q_t. \tag{44}$$

A.2 Solution to HH Problem

A.2.1 Proof of Proposition 1, Part 1

We define detrended variables (divided by Z_t) $\hat{C}_{t,i}$, $\hat{W}_{t,i}$, $\hat{S}_{t+1,i}$, $\hat{B}^H_{t+1,i}$, and $\hat{q}^h_t$. We further define the value function $\hat{V}^H(W_{t,i},\mathcal{Z}_t)/Z_t$: Denoting household wealth at the beginning of the period as $W_{t,i}$, the recursive problem is

$$\hat{V}^H(\hat{W}_{t,i},\mathcal{Z}_t) =$$
$$\max_{\substack{\hat{S}_{t+1,i},\hat{B}^H_{t+1,i},H^L_{t+1,i},\\ \hat{H}^O_{t+1,i},\hat{C}_{t,i},\eta_{t,i}}} \left((1-\beta)\left(\hat{C}_{t,i}+\psi\hat{S}_{t+1,i}\sqrt{\eta_{t,i}}\right)^{1-\gamma} + \beta\mathrm{E}_t\left[\exp((1-\sigma)g_{t+1})\hat{V}^H(\hat{W}_{t+1,i},\mathcal{Z}_{t+1})^{1-\sigma}\right]^{\frac{1-\gamma}{1-\sigma}}\right)^{1/(1-\gamma)} \tag{45}$$

subject to the budget constraint

$$\hat{W}_{t,i} + \eta_{t,i}\hat{f}_t H^O_{t,i} \geq \hat{C}_{t,i} + Q_t\hat{S}_{t+1,i} + p_t\hat{B}^H_{t+1,i} + \hat{q}^L_t H^L_{t+1,i} + \hat{q}^O_t H^O_{t+1,i}, \tag{46}$$

the transition law for wealth

$$\begin{aligned}\hat{W}_{t+1,i} =& \exp(-g_{t+1})(D_{t+1}+Q_{t+1})n^\eta_{t+1,i}\hat{S}_{t+1,i} + (\hat{w}_{t+1}+\hat{q}^L_{t+1})\epsilon^H_{t+1,i}H^L_{t+1,i}\\ &+ \hat{q}^O_{t+1}\epsilon^F_{t+1,i}H^O_{t+1,i} + \exp(-g_{t+1})\hat{B}^H_{t+1,i},\end{aligned} \tag{47}$$

and no-shorting conditions for both assets.

Note that given the permanent effect on wealth of idiosyncratic shocks, households generally have different levels of wealth given their individual history of shocks. However, the we conjecture and verify that the value function is homogeneous of degree one in wealth:

$$\hat{V}^H(\hat{W}_{t,i}, \mathcal{Z}_t) = v^H(\mathcal{Z}_t)\hat{W}_{t,i}.$$

Using lower case letters to denote ratios with respect to wealth $\hat{W}_{t,i}$, we can write the problem as

$$v_t^H \hat{W}_{t,i} = \max_{\substack{\hat{s}_{t+1}, \hat{b}^H_{t+1}, h^L_{t+1}, \\ h^O_{t+1}, \eta_t, \hat{c}_t}} \left(\hat{W}_{t,i}^{1-\gamma}(1-\beta)\left(\hat{c}_t + \psi \hat{s}_{t+1}\sqrt{\eta_t}\right)^{1-\gamma} + \hat{W}_{t,i}^{1-\gamma}\beta \mathrm{E}_t\left[\exp((1-\sigma)g_{t+1})(v^H_{t+1} r^H_{t+1,i})^{1-\sigma}\right]^{\frac{1-\gamma}{1-\sigma}}\right)^{1/(1-\gamma)} \tag{48}$$

subject to the budget constraint

$$1 + \eta_{t,i}\hat{f}_t h_t^O \geq \hat{c}_t + Q_t \hat{s}_{t+1} + p_t \hat{b}^H_{t+1} + \hat{q}^L_t h^L_{t+1} + \hat{q}^O_t h^O_{t+1}, \tag{49}$$

the transition law for the growth rate of wealth

$$\begin{aligned} \hat{r}^H_{t+1,i} = &\exp(-g_{t+1})(D_{t+1} + Q_{t+1}) n^\eta_{t+1,i}\hat{s}_{t+1} + (\hat{w}_{t+1} + \hat{q}^L_{t+1})\epsilon^H_{t+1,i} h^L_{t+1} \\ &+ \hat{q}^O_{t+1}\epsilon^F_{t+1,i} h^O_{t+1} + \exp(-g_{t+1})\hat{b}^H_{t+1}, \end{aligned} \tag{50}$$

and no-shorting conditions for all assets, where we defined the gross return on household wealth $\hat{r}^H_{t+1,i} = \hat{W}_{t+1,i}/\hat{W}_{t,i}$.

Wealth $\hat{W}_{t,i}$ cancels on both sides of equation (48), which verifies the conjecture. This implies that we can solve the problem of a representative household, since all households choose the same *ratios* of consumption and portfolio positions relative to wealth. Households differ in their *levels* of wealth, yet we can use the homogeneity of the optimization problem in wealth to obtain aggregation.

Certainty Equivalent and its Derivative Denote $\hat{V}(\hat{W}_{t,i}, \mathcal{Z}_t) \equiv \hat{V}^H_t$ and the certainty equivalent:

$$\mathrm{CE}_t = \mathrm{E}_t\left[\exp((1-\sigma)g_{t+1})(v^H_{t+1}\hat{r}^H_{t+1,i})^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{51}$$

We compute the partial derivative of the value function term containing the certainty equivalent with respect to the next-period value function

$$\begin{aligned} \frac{\partial}{\partial v^H_{t+1}} \mathrm{E}_t\left[\exp((1-\sigma)g_{t+1})(v^H_{t+1}\hat{r}^H_{t+1,i})^{1-\sigma}\right]^{\frac{1-\gamma}{1-\sigma}} &= \frac{\partial \mathrm{CE}_t^{1-\gamma}}{\partial v^h_{t+1}} \\ &= (1-\gamma)\exp((1-\sigma)g_{t+1})(v^H_{t+1})^{-\sigma}(\hat{r}^H_{t+1,i})^{1-\sigma}\mathrm{CE}_t^{\sigma-\gamma}. \end{aligned} \tag{52}$$

Similarly, the derivative with respect to wealth growth is

$$\begin{aligned} \frac{\partial}{\partial \hat{r}^H_{t+1,i}} \mathrm{E}_t\left[\exp((1-\sigma)g_{t+1})(v^H_{t+1}\hat{r}^H_{t+1,i})^{1-\sigma}\right]^{\frac{1-\gamma}{1-\sigma}} &= \frac{\partial \mathrm{CE}_t^{1-\gamma}}{\partial \hat{r}^H_{t+1,i}} \\ &= (1-\gamma)\exp((1-\sigma)g_{t+1})(v^H_{t+1})^{1-\sigma}(\hat{r}^H_{t+1,i})^{-\sigma}\mathrm{CE}_t^{\sigma-\gamma}. \end{aligned} \tag{53}$$

Envelope Condition The envelope condition is

$$\frac{\partial \hat{V}^H(\hat{W}_{t,i}, \mathcal{Z}_t)}{\partial \hat{W}_{t,i}} = v_t^H = \mu_t^H, \tag{54}$$

where we attach the Lagrange multiplier μ_t^H to the budget constraint.

First-order condition for consumption The consumption FOC is

$$\mu_t^H = (1-\beta)u_t^{-\gamma}(v_t^H)^{\gamma}, \tag{55}$$

where $u_t = \hat{c}_t + \psi \hat{s}_{t+1}\sqrt{\eta_t}$.

SDF The household's intertemporal marginal rate of substitution between time t and $t+1$ is

$$\begin{aligned}
M_{t,t+1}^i = \frac{\frac{\partial V_t^H}{\partial C_{t+1,i}}}{\frac{\partial V_t^H}{\partial C_{t,i}}} &= \frac{\frac{\partial v_t^H}{\partial \hat{c}_{t+1}}}{\frac{\partial v_t^H}{\partial \hat{c}_t}} \frac{\exp(-g_{t+1})}{\hat{r}_{t+1,i}^H} = \frac{\partial v_t^H}{\partial v_{t+1}^H} \frac{\exp(-g_{t+1})}{\hat{r}_{t+1,i}^H} \frac{\partial v_{t+1}^H / \partial \hat{c}_{t+1}}{\partial v_t^H / \partial \hat{c}_t} \\
&= \beta \exp(-\sigma g_{t+1})(v_{t+1}^H)^{-\sigma}(\hat{r}_{t+1,i}^H)^{-\sigma} \mathrm{CE}_t^{\sigma-\gamma}(v_t^H)^{\gamma} \frac{v_{t+1}^H}{v_t^H} \\
&= \beta \exp(-\sigma g_{t+1}) \left(\frac{v_{t+1}^H}{v_t^H}\right)^{1-\gamma} \left(\frac{v_{t+1}^H}{\mathrm{CE}_t}\right)^{\gamma-\sigma} (\hat{r}_{t+1,i}^H)^{-\sigma}.
\end{aligned} \tag{56}$$

The first line is using the fact that

$$\frac{\partial V_t^H}{\partial C_{t+1,i}} = \frac{\partial v_t^H}{\partial \hat{c}_{t+1}} \frac{\partial V_t^H}{\partial v_t^H} \frac{\partial \hat{c}_{t+1}}{\partial C_{t+1,i}} = \frac{\partial v_t^H}{\partial \hat{c}_{t+1}} \frac{Z_t \hat{W}_t}{Z_{t+1}\hat{W}_{t+1}} = \frac{\partial v_t^H}{\partial \hat{c}_{t+1}} \frac{\exp(-g_{t+1})}{\hat{r}_{t+1,i}^H}.$$

The second line uses

$$\frac{\partial v_t^H}{\partial \hat{c}_t} = \mu_t^H = v_t^H,$$

by equations (54) and (55), and the derivative in (52).

A.2.2 First-order Conditions

FOC for bonds The FOC for bonds is

$$0 = -\mu_t^H p_t + \frac{(v_t^H)^{1/(1-\gamma)-1}}{1-\gamma} \mathrm{E}_t \left[\beta \frac{\partial \hat{r}_{t+1,i}^H}{\partial \hat{b}_{t+1}^H} \frac{\partial \mathrm{CE}_t^{1-\gamma}}{\partial \hat{r}_{t+1,i}^H}\right].$$

Noting that

$$\frac{\partial \hat{r}_{t+1,i}^H}{\partial \hat{b}_{t+1}^H} = \exp(-g_{t+1}),$$

the HH Euler equation for bonds is

$$p_t = \mathrm{E}_t\left[M^i_{t,t+1}\right], \tag{57}$$

using the derivative in (53) and the definition of the SDF (18).

FOC for equity The FOC for equity is

$$0 = -\mu^H_t \hat{Q}_t + (1-\beta)\psi u_t^{-\gamma}\sqrt{\eta_t}(v^H_t)^{\gamma} + \frac{(v^H_t)^{1/(1-\gamma)-1}}{1-\gamma}\mathrm{E}_t\left[\beta\frac{\partial \hat{r}^H_{t+1,i}}{\partial s_{t+1}}\frac{\partial \mathrm{CE}^{1-\gamma}_t}{\partial \hat{r}^H_{t+1,i}}\right].$$

Given

$$\frac{\partial r^H_{t+1,i}}{\partial s_{t+1}} = \exp(-g_{t+1})(D_{t+1}+Q_{t+1})n^{\eta}_{t+1,i},$$

the Euler equation for equity is

$$\hat{Q}_t = \psi\sqrt{\eta_t} + \mathrm{E}_t\left[M^i_{t,t+1}(D_{t+1}+Q_{t+1})n^{\eta}_{t+1,i}\right]. \tag{58}$$

which we can write as

$$\hat{Q}_t = \psi\sqrt{\eta_t} + \mathrm{E}_t\left[M^i_{t,t+1}v^F(\mathcal{Z}_{t+1})\left(\left(1+\eta(\epsilon^F_{t+1,i}-1)\right)k_{t+1}R^K_{t+1} - (1-\tau^{int})b_{t+1}\right)\right]. \tag{59}$$

FOC for labor human capital The FOC for human capital is

$$0 = -\mu^H_t \hat{q}^L_t + \frac{(v^H_t)^{1/(1-\gamma)-1}}{1-\gamma}\mathrm{E}_t\left[\beta\frac{\partial \hat{r}^H_{t+1,i}}{\partial h^L_{t+1}}\frac{\partial \mathrm{CE}^{1-\gamma}_t}{\partial \hat{r}^H_{t+1,i}}\right].$$

Given

$$\frac{\partial \hat{r}^H_{t+1,i}}{\partial h^L_{t+1}} = (\hat{q}^L_{t+1}+\hat{w}_{t+1})\epsilon^H_{t+1,i},$$

the Euler equation for human capital is

$$\hat{q}^L_t = \mathrm{E}_t\left[M^i_{t,t+1}\exp(g_{t+1})(\hat{q}^L_{t+1}+\hat{w}_{t+1})\epsilon^H_{t+1,i}\right]. \tag{60}$$

FOC for managerial human capital The FOC for managerial human capital is

$$0 = -\mu^H_t(\hat{q}^O_t - \hat{f}_t\eta_t) + \frac{(v^H_t)^{1/(1-\gamma)-1}}{1-\gamma}\mathrm{E}_t\left[\beta\frac{\partial \hat{r}^H_{t+1,i}}{\partial h^O_{t+1}}\frac{\partial \mathrm{CE}^{1-\gamma}_t}{\partial \hat{r}^H_{t+1,i}}\right].$$

Given

$$\frac{\partial \hat{r}^H_{t+1,i}}{\partial h^O_{t+1}} = \hat{q}^O_{t+1},$$

the Euler equation for managerial human capital is

$$\hat{q}^O_t = \hat{f}_t\eta_t + \mathrm{E}_t\left[M^i_{t,t+1}\exp(g_{t+1})\hat{q}^O_{t+1}\right]. \tag{61}$$

FOC for η The FOC for concentration η_t is

$$0 = \mu_t^H h_{t+1}^O \hat{f}_t + (1-\beta)\frac{\psi}{2} u_t^{-\gamma} \hat{s}_{t+1} \eta_t^{-1/2} (v_t^H)^\gamma + \frac{(v_t^H)^{1/(1-\gamma)-1}}{1-\gamma} \mathrm{E}_t \left[\beta \frac{\partial \hat{r}_{t+1,i}^H}{\partial \eta_t} \frac{\partial \mathrm{CE}_t^{1-\gamma}}{\partial \hat{r}_{t+1,i}^H} \right].$$

Given

$$\frac{\partial \hat{r}_{t+1,i}^H}{\partial \eta_t} = \hat{s}_{t+1} \exp(-g_{t+1}) v^F(\mathcal{Z}_{t+1})(\epsilon_{t+1,i}^F - 1) k_{t+1} R_{t+1}^K,$$

the Euler equation for η_t is

$$h_{t+1}^O \hat{f}_t + \frac{\psi \hat{s}_{t+1}}{2\sqrt{\eta_t}} = \hat{s}_{t+1} \mathrm{E}_t \left[M_{t,t+1}^i v^F(\mathcal{Z}_{t+1})(1 - \epsilon_{t+1,i}^F) k_{t+1} R_{t+1}^K \right]. \tag{62}$$

A.2.3 Proof of Proposition 1, Part 2

Owning a claim to one unit of the equity fund (corresponding to one dollar of firm net worth) entitles household i to receive dividend payment

$$D_t = \xi_0 - x_t - \frac{\xi_1}{2} x_t^2.$$

If households re-invest this dividend in the fund perpetually, we can compute the value of this claim to households recursively as

$$v^\eta(\mathcal{Z}_t) = D_t + \mathrm{E}_t \left[M_{t,t+1}^i n_{t+1,i}^\eta v^\eta(\mathcal{Z}_{t+1}) \right], \tag{63}$$

since net worth in the fund grows at rate $n_{t+1,i}^\eta$ based on the optimal decisions of firms. This implies that the ex-dividend market price of this equity claim is

$$Q_t^\eta = \mathrm{E}_t \left[M_{t,t+1}^i n_{t+1,i}^\eta v^\eta(\mathcal{Z}_{t+1}) \right].$$

We are looking for the SDF $\mathcal{M}_{t,t+1}^i$ that households impose on firm i – the firm that has concentrated ownership from fund i – such that the value of the firm as defined in (35) is equal to household i's valuation in (63). Inspection of (35) and (63) immediately reveals that

$$\mathcal{M}_{t,t+1}^i = M_{t,t+1}^i \frac{n_{t+1,i}^\eta}{n_{t+1,i}}$$

achieves this goal.

B Computational Methods

B.1 Numerical Integration of Idiosyncratic Shocks

Several equilibrium objects contain expectations over functions of idiosyncratic shocks $\boldsymbol{\epsilon}_t = (\epsilon^H_{i,t}, \epsilon^F_{i,t})$. In particular, recall that the HH SDF is

$$M^i_{t,t+1} = \beta \exp(-\sigma g_{t+1}) \left(\frac{v^H_{t+1}}{v^H_t}\right)^{1-\gamma} \left(\frac{v^H_{t+1}}{\mathrm{CE}_t}\right)^{\gamma-\sigma} (\hat{r}^H_{t+1,i}(\boldsymbol{\epsilon}_{t+1}))^{-\sigma},$$

where the growth rate of wealth is

$$\hat{r}^H_{t+1,i}(\boldsymbol{\epsilon}_{t+1}) = \exp(-g_{t+1}) v^F(\mathcal{Z}_{t+1}) n^\eta_{t+1,i} \hat{s}_{t+1} + (\hat{w}_{t+1} + \hat{q}^h_{t+1}) \epsilon^H_{t+1,i} h_{t+1} + \exp(-g_{t+1}) \hat{b}^H_{t+1}.$$

Further recalling that the growth rate of firm net worth is

$$n^\eta_{t+1,i} = \left(1 + \eta(\epsilon^F_{t+1,i} - 1)\right) k_{t+1} R^K_{t+1} - (1 - \tau^{int}) b_{t+1},$$

we can see that the SDF depends on both components of $\boldsymbol{\epsilon}_t$. Idiosyncratic shocks terms further appear in the certainty equivalent (51), the FOC for equity (59), and the FOC for human capital (60).

The four integrals to be computed for these conditions are $\mathrm{E}_{\boldsymbol{\epsilon}}[\hat{r}^H_{t+1}(\boldsymbol{\epsilon})^{-\sigma}]$, $\mathrm{E}_{\boldsymbol{\epsilon}}[\epsilon^H \hat{r}^H_{t+1}(\boldsymbol{\epsilon})^{-\sigma}]$, $\mathrm{E}_{\boldsymbol{\epsilon}}[\epsilon^F \hat{r}^H_{t+1}(\boldsymbol{\epsilon})^{-\sigma}]$ and $\mathrm{E}_{\boldsymbol{\epsilon}}[\hat{r}^H_{t+1}(\boldsymbol{\epsilon})^{1-\sigma}]$.

To compute the necessary integrals, we discretize the cross-sectional joint distributions of shocks producing a $2 \times K$ matrix of nodes and a $1 \times K$ vector of probability weights for each pair of nodes. There are two such joint distributions – one in normal times ($d_t = 0$) and another in disasters ($d_t = 1$).

No Disasters In normal times, the two shocks are jointly log-normally distributed with mean $\boldsymbol{\mu} = (\mu^h, \mu^e)$ and co-variance matrix

$$\Sigma = \begin{bmatrix} \sigma^2_H & \sigma_{H,E} \\ \sigma_{H,E} & \bar{\sigma}^2_E \end{bmatrix}.$$

where $\sigma_{H,E} = \rho_{H,E} \sigma^H \bar{\sigma}^E$.

This implies that $\log(\boldsymbol{\epsilon})$ is jointly normally distributed with mean $\hat{\boldsymbol{\mu}} = (\hat{\mu}_H, \hat{\mu}_E)$ and co-variance matrix

$$\hat{\Sigma} = \begin{bmatrix} \hat{\sigma}^2_H & \hat{\sigma}_{H,E} \\ \hat{\sigma}_{H,E} & \hat{\bar{\sigma}}^2_E \end{bmatrix},$$

where

$$\hat{\sigma}^2_j = \log\left(1 + \left(\frac{\sigma^j}{\mu^j}\right)^2\right),$$

$$\hat{\mu}_j = \log(\mu_j) - \frac{\hat{\sigma}^2_j}{2},$$

for $j = H, E$ and the covariance is

$$\hat{\sigma}_{H,E} = \log\left(1 + \frac{\sigma_{he}}{\mu_H \mu_E}\right).$$

We normalize $\mu_H = \mu_E = 1$. To calculate the integrals numerically, we discretize each marginal distribution using Gaussian Quadrature and construct the joint distribution using the change of variables technique of Judd (1998) pp. 276-277.

2D Gaussian Quadrature To do so, we first compute the Cholesky decomposition $\hat{\Omega}$ of the covariance matrix, such that $\hat{\Sigma} = \hat{\Omega}\hat{\Omega}'$. Then we define the transformation T for any bivariate column vector $\boldsymbol{\nu}$

$$T(\boldsymbol{\nu}) = \exp\left(\sqrt{2}\hat{\Omega}\boldsymbol{\nu} + \hat{\boldsymbol{\mu}}'\right).$$

We use K_j quadrature nodes for dimension $j = h, e$, respectively, denoting the set of nodes and weights in each dimension by

$$\{\nu_n^j, \omega_n^j\}_{n=1}^{K_j}.$$

Relying on standard results, we can then approximate the expectation over a smooth function $G(\boldsymbol{\epsilon})$ as

$$\mathrm{E}_{\boldsymbol{\epsilon}}\left[G(\boldsymbol{\epsilon})\right] = \int_{\mathrm{supp}(\boldsymbol{\epsilon})} G(\boldsymbol{\epsilon})\, dF(\boldsymbol{\epsilon}) = \frac{1}{\pi} \sum_{\substack{n=1,\dots,K_h \\ m=1,\dots,K_e}} G\left(T\left([\nu_n^h,\, \nu_m^e]'\right)\right) \omega_n^h \omega_m^e.$$

We set $K_h = K_e = 7$, which yields $K = 49$ nodes (each consisting of a value for both shocks) and probability weights.

Disasters The joint distribution of human capital and firm shocks in disasters is a mixture of two jointly normal distributions. The marginal distribution of firm shocks is the same in both components of the mixture: it is normal with mean $\hat{\mu}^e$ and standard deviation $\hat{\hat{\sigma}}^E + \zeta^\sigma$.

The marginal distribution of human capital shocks is a mixture of two normal distributions denoted as "low" (L) and "high" (H) with mixing probability of $\frac{1}{2}$. In other words, in disasters

$$\epsilon_i^H = \mathbb{1}_{1/2}\epsilon_i^{H,low} + (1 - \mathbb{1}_{1/2})\epsilon_i^{H,high}$$

where $\mathbb{1}_{1/2}$ is a single Bernoulli random variable with probability $\frac{1}{2}$, $\log \epsilon_i^{H,low} \sim \mathcal{N}(\hat{\mu}_H - \Delta, \hat{\sigma}_{H,low})$ and $\log \epsilon_i^{H,high} \sim \mathcal{N}(\hat{\mu}_H + \Delta, \hat{\sigma}_H)$ with $\Delta \geq 0$ without loss of generality. We keep the standard deviation of the "high" distibution at $\hat{\sigma}_H$, same as the standard deviation of the normally distributed shocks in good times, and we select values of Δ and $\hat{\sigma}_{H,low}$ to match scaled non-parametric moments of labor income shocks documented by Guvenen, Ozkan, and Song (2014) – 90th percentile minus median, and median minus 10th percentile. The percentiles are scaled following Appendix C.2.

Covariance matrices of the joint distributions are implied by correlations. The "high" joint distribution inherits the same correlation between human capital and firm components as the no-disaster correlation $\rho_{H,E}$. The "low" joint distribution has a correlation $\hat{\rho}^{low}_{H,E}$ chosen such that the correlation coefficient of the mixture equals the correlation of log shocks in the no-disasters periods $\hat{\rho}_{H,E}$ (which in turn implies a shock correlation of $\rho_{H,E}$):

$$\hat{\rho}^{low}_{H,E} = \rho_{H,E}\frac{\sigma_{H,mix} - \sigma_H/2}{\sigma_{H,low}/2}, \text{ where}$$
$$\sigma^2_{H,mix} = \frac{1}{2}\left(\sigma^2_H + \sigma^2_{H,low} - \Delta^2\right)$$

We discetize both joint distributions using the same method as before – 2D Gaussian Quadrature – producing 49 nodes and weights each. The mixture then has $K = 98$ nodes and weights, where the weights are multiplied by mixing probability $\frac{1}{2}$ to ensure that they sum to 1.

C Data Appendix

C.1 Externally Calibrated Parameters

In this section we detail our calibration strategy for parameters listed in Table 2.

Several parameters are directly set to easily measured or standard values commonly used in the literature, listed in Table 2. We annualize all parameters and simulated model moments where appropriate unless otherwise noted.

Functional forms. The investment technology is given by

$$\Phi(I_t/\bar{K}_t) = \frac{\phi}{2}\left(\frac{I_t}{\bar{K}_t} - \tilde{\delta}\right)^2, \tag{64}$$

where $\tilde{\delta} = \delta + \exp(\bar{g}) - 1$ to allow for investment offsetting trend growth without incurring adjustment costs. The parameter ϕ governs the strength of the adjustment cost friction. Similarly, the equity issuance cost is given by

$$\Xi(X_t/N_t) = \frac{\xi_1}{2}\left(\frac{X_t}{N_t}\right)^2, \tag{65}$$

where parameter ξ_1 determines the magnitude of the cost. It is easy to show that the case of $\xi_1 = 0$ implies that the equity value of a firm is equal to its net worth, i.e. $V^F(N_{t,i}, \mathcal{Z}_t) = N_t$. However, with $\xi_1 > 0$ these values differ.

Growth and Disasters. Trend productivity growth $\bar{g}$ is set to 1.8% based on average growth in labor productivity. We set the quarterly probability of entering into a disaster at $\pi^d = 0.62\%$, and the quarterly probability of remaining in the disaster state is $\pi^s = 70\%$. These probabilities imply an unconditional quarterly probability of being in the disaster state of 2% (annual 2.82%), in line the empirical findings of Nakamura, Steinsson, Barro, and Ursúa (2013). The expected length of a disaster is 4 quarters; our disasters are significantly less persistent than in Nakamura et al. (2013). We further set the permanent component of the productivity decrease in disasters to a fraction $\zeta^p = 0.05$ of the transitory TFP drop. One aspect of disasters unique to our model is that idiosyncratic business as well as human capital risk rise in disaster periods. In particular, relative to its baseline value in normal times to be estimated below, idiosyncratic capital shock dispersion rises by $\zeta^\sigma = 20.00\%$ in disasters. While it is difficult to measure this rise in dispersion directly in the data, it is consistent with the spike in idiosyncratic volatility of stock returns during the Great Depression documented in Figure 2 of Section 2. Idiosyncratic human capital shocks are distributed log-normally in non-disaster states, but log-realizations are drawn from a normal-mixture distribution during disasters. Using the mixture distribution, we calibrate a spike in negative skewness with magnitude based on the evidence in Guvenen, Ozkan, and Song (2014): Kelly's skewness κ_t^H declines to $\kappa_t^H = \underline{\kappa}^H = -0.21$ during disasters. [28]

Firms. Technology parameters are set to standard values, with the Cobb-Douglas coefficient on labor targeting a labor share of 0.667, and annual capital depreciation set to 7.81 in line with BEA investment data. The corporate profit tax rate is set to the average faced by corporations based on current U.S. tax code, including a 21% Federal tax rate and an average of state-level taxes. To capture the magnitude of interest tax deductibility, we compute the average spread between the Moody's AAA corporate bond index yield and our measure of the short-term risk free rate (the 3-month Tbill rate), which is 0.68% per quarter. We then set τ^{int} to the product of 25% tax rate with the effective debt tax rate given by risk-free rate plus spread.

Households. We set the standard deviation of log human capital shocks σ_H to 9.21%. We derive this number by transforming an estimate for the standard deviation of persistent yet transitory earnings shocks into our i.i.d. shocks to the stock of human capital, with details on this transformation in Appendix C.2. The i.i.d. shocks to human capital are effectively permanent shocks to earnings. Our transformation aims to compute the standard deviation of permanent shocks equivalent to mean-reverting persistent shocks by equating the present value of either shock innovation. All other parameters related to households are estimated jointly in Section 4.2 below. We set the correlation between labor human capital and firm shocks, $\rho_{H,E}$, to 0.125 based on a simple calculation in the 2019 SCF data described in Section 2. In particular, we compute the aggregate correlation of labor income with equity ownership – which fraction of labor income does the representative household earn at a firm, in which they hold an undiversified equity stake? We calculate this number for each decile as product of the private equity share (solid orange line in left plot of Figure 1) with the income share earned by business owners (solid orange line in the right plot of Figure 1). We then compute the weighted average of these products, using as weights the equity-income share of each decile, i.e. the product of blue bars on the left with blue bars on the right. This calculation yields a correlation of 7.2%. We perform the analogous calculation for employees of public firms – using the dashed black lines in each graph, yielding an additional correlation of 2.0%. Adding these numbers gives a total of 9.2%, which likely serves as a lower bound for the correlation of labor income with

[28] The assumption that these idiosyncratic risks only rise during disasters and not during regular business cycle downturns is a conservative calibration choice. Kelly's skewness of log human capital shocks is zero in normal times.

undiversified business risk at the micro level. We will discuss the effect of this correlation on our results below.

C.2 Human Capital Shocks

We calibrate shocks to human capital using data on variation in individual labor income. We follow Favilukis (2013) in assuming that persistent labor income shocks have a volatility of $\tilde{\sigma}_H = 13.5\%$ and a persistence of $\rho_H = 0.95$. At a constant discount rate of r_H, the effect of a one-sd shock to the value of human capital is given by the present value

$$\sum_{t=0}^{\infty} \frac{\exp(\tilde{\sigma}_H(\rho_H)^t)}{(1+r^h)^t}$$

which therefore implies the standard deviation of the equivalent shock to human capital. A challenge is that the appropriate rate r^h at which to discount human capital depends on the stochastic discount factor of the household, which in turn depends both on the amount of exogenous idiosyncratic and endogenous aggregate risk she faces. We solve this fixed point problem at the steady state of the model i.e., given the labor income shock process parametrized by $\tilde{\sigma}_H$ and ρ^h, we find the dispersion of iid human capital shocks σ_H that satisfies the present value calculation at the model steady-state discount rate for human capital consistent with a σ_H amount of risk. This calculation produces $\sigma_H = 9.21\%$.

D Results Appendix

D.1 Effects of Financial Frictions

Financial frictions in the shape of equity issuance costs and the leverage constraint are key features that allow the model to match the high volatility of equity excess returns while simultaneously generating the low volatility of corporate payouts in the data. Table A.1 highlights the effects of these frictions by comparing the calibrated baseline to two other economies in which financial frictions are tuned off. First, in the 2nd column, we simply eliminate equity issuance costs by setting $\xi_1 = 0$. In this model, firms still face a leverage constraint, but households can freely adjust firm equity without any costs. When raising new equity is costless, firms choose to be at their binding leverage constraint 100% of the time. In contrast, when equity issuance costs are as large as in the baseline, firms never exhaust their leverage constraint. To make the models comparable, we therefore also adjust the maximum leverage parameter in the $\xi_1 = 0$ to 40% (recall this parameter has a value of 70% in the baseline). Given this difference in maximum leverage, the two models produce roughly the same amount of observed leverage.

A binding leverage constraint and costless deviations from the firm's payout target cause much more volatile dividend growth. While the baseline matches the 7.1% volatility in the data, the counterfactual model with $\xi_1 = 0$ generates 18.8% volatility.

Turning to asset prices, the average equity premium declines only moderately, by less than 1pp, when financial frictions disappear. However, the volatility of excess returns drops substantially from the data value of 16.2% to only 13.2%. Thus, the model without financial

Table A.1: Effects of Financial Frictions

	Baseline	$\xi_1 = 0$	$\xi_1 = 0$ **& exog. lev.**
Leverage, mean	32.4	39.9	39.9
Constraint binds, mean	13.4	100.0	–
Dividend gr, vol	7.11	18.79	18.29
vol(Firm value / Net worth)	3.80	0.00	0.00
Market Excess Return, mean	6.17	7.15	7.34
Market Excess Return, vol	16.24	13.17	13.40
Risk Share, Portfolio	34.0	50.2	48.0
Risk Share, Private	76.0	83.7	83.5

frictions greatly overstates the Sharpe ratio of the market. Taken together, the model without equity issuance costs is a victim of the excess volatility puzzle: corporate payouts are much too volatile, yet returns are too stable. How do financial frictions boost the volatility of returns while dampening that of dividends? They do so by introducing a volatile wedge between the value of firm equity and firm net worth. In the model with $\xi_1 = 0$, these two values are by construction the same, and thus the volatility of the ratio of firm value / net worth is zero. In the baseline with $\xi_1 = 11$, however, the (annual) volatility of this ratio is 7.8%.

In the 3rd column of Table A.1 we verify that the presence of the collateral constraint on its own has little effect on firm payouts and returns, relative to a model in which leverage is fixed exogenously at 40% each period. Simulated moments for the model with $\xi_1 = 0$ and exogenous leverage are almost identical to those in the $\xi_1 = 0$ economy of column 2.

Since the model without financial frictions cannot match *aggregate* excess return volatility in the data, it overstates the importance of idiosyncratic risk in the $\tilde{\eta} = \eta_t$ and $\tilde{\eta} = 1$ portfolios: the idiosyncratic variance shares of these portfolios would be 50.2% and 83.7%, respectively, which is about 10pp higher than the data. Put differently, in the model without financial frictions, our inference about the relative magnitude of idiosyncratic firm shocks $\bar{\sigma}_E$ and the source of non-diversification α_O and ψ would be misled by a lack of aggregate risk.

D.2 Macro Dynamics

Here, we document the dynamics of macroeconomic aggregates and asset prices in response to modeled shocks. We first consider the behavior of the economy in disasters before turning to increases in the expected severity of a disaster *absent* its realization. Figure A.1 plots a generalized impulse response function to a moderate and large disaster shock, respectively. The figure shows that disasters are periods of massive yet short-lived declines in output, consumption and investment, with larger declines in productivity leading to bigger contractions. The response of the risk-free rate depends on qualitatively on the disaster severity. The strong consumption smoothing motive pushing rates up in disasters is counter-acted by a motive to save as a precaution in moderate disasters against the risk of the disaster turning more severe.

Time-varying fear of these unlikely – the quarterly probably of entering into the disaster state

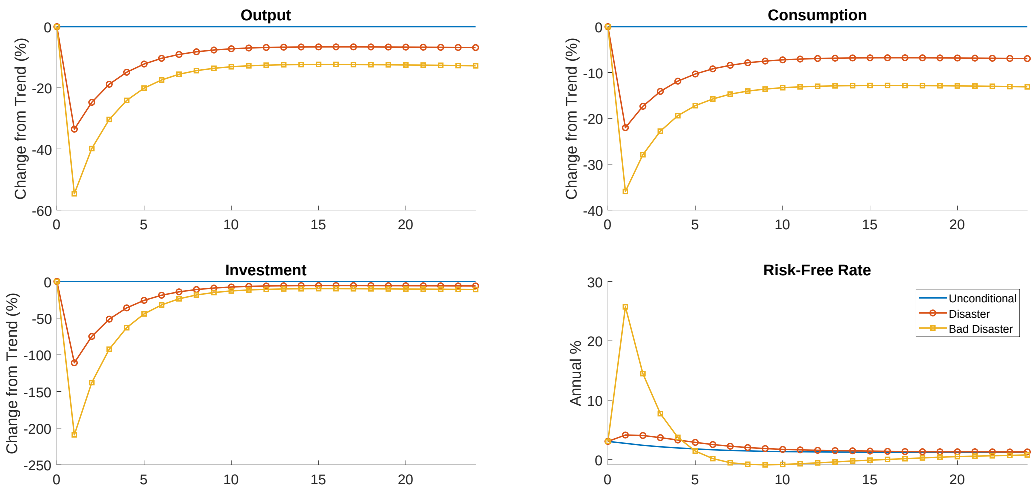


Figure A.1: Disaster Impulse Response

is 0.62% – but large macro risks drives variation in discount rates outside of disasters. Figure A.2 shows the response of the same macro aggregates as in Figure A.1 to a spike in disaster severity from moderate to large. This shock triggers a massive increase in precautionary savings demand, pushing interest rates into negative territory and causing strong substitution from consumption to investment. Since realized productivity is not affected, the impact on output is minimal.

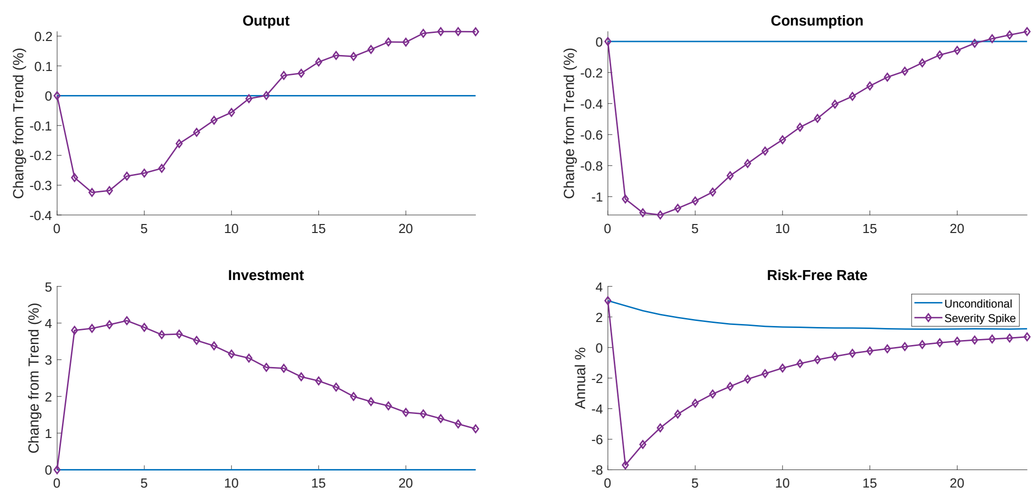


Figure A.2: Spike in Disaster Severity: Macro Variables